Chapter 5
Design Response Spectra and Acceleration Time Histories

5-1. Defining the Design Earthquake

In a linear-elastic response spectrum analysis, response spectra define the free field ground motion for the design earthquake. A response spectrum gives the maximum damped response (expressed as displacement, velocity, or acceleration) of all possible linear single degree-of-freedom systems using the natural frequency (or period) to describe the system. Viscous damping expressed as a percentage of critical damping is used to develop a response spectra. A design earthquake is often defined by a set of response spectra for various damping ratios. The response spectra produced by recorded earthquake events are characterized by a jagged shape made up of peaks and valleys of varying magnitude; however, design response spectra are smoothed so that they are not frequency sensitive.

5-2. Developing Design Response Spectra

a. Deterministic and probabilistic approaches. Design response spectra are developed by using either a "deterministic approach" or a "probabilistic approach." The probabilistic approach is based on probabilistic seismic hazard analysis methodology which in essence uses the same elements as the deterministic approach, but adds an assessment of the likelihood that ground motion will occur during a specified time period.

b. Procedures. There are two basic procedures for developing design response spectra using either the deterministic or probabilistic approach. They are: (1) anchoring the spectral shape to the peak ground acceleration; and (2) estimating the spectrum directly. Although procedure (1) is more often used, the use of procedure (2) is increasing, and for some situations is preferred because it incorporates factors besides just the local site conditions.

c. Obtaining design response spectra. It is beyond the scope of this EP to present the detailed procedures for developing design response spectra, or for forecasting PGA's for design earthquakes. Refer to ETL 1110-2-301, ETL 1110-2-303, and "Tentative Provisions for the Development of Seismic Regulations for Buildings" (Applied Technology Council 1984) for further information on developing design response spectra to define the design earthquakes.

5-3. Developing Acceleration Time Histories

a. Matching design response spectrum. The more refined methods of analysis discussed in paragraph 2-2*d* are of the time-history type. Time histories usually express the ground motion as a record of acceleration with respect to time. Acceleration time histories should be developed so their response spectrum is consistent with the previously established site-specific design response spectrum described in paragraph 5-5*c*. The time histories should also have a strong motion duration appropriate to the particular design earthquake.

b. Procedures. There are two basic procedures for developing acceleration time histories: (1) selecting a suite of past recorded earthquake ground motions, and (2) synthetically developing or modifying one or more ground motions.

(1) When selecting a suite of time-history records for the first procedure, the intent is to cover the valleys of the spectrum produced by one record, which fall significantly below the site-specific design response spectrum, with better matching spectral values at these frequencies as produced by the other records in the suite. It is also necessary that the spectra produced by the suite of records not significantly exceed the site-specific design response spectrum. Primary advantage of this procedure is that the structure is analyzed by real, natural ground motions that are representative of what the structure could experience.

(2) When using the second procedure, it is possible to either completely synthesize an accelerogram, or modify an actual recorded earthquake accelerogram so that the response spectrum of the resultant accelerogram closely fits or matches the site-specific design response spectrum. The primary advantage of this procedure is that a good fit to the design response spectrum can be achieved with a single accelerogram, thus only a single dynamic analysis is required.

5-4. Dynamic Analysis by Modal Superposition

a. Frequencies and mode shapes. The linear-elastic response spectrum method utilizes modal superposition dynamic analysis to determine the structural response.

b. Time-history analysis. Once the modes are derived, the response of the complex multiple degree-of-freedom system is reduced to the solution of the simple, single basic equation of motion for a single degree-of-freedom (SDOF) system. For time-history analysis, the response is easily obtained using step-by-step integration of the equation of motion for the SDOF system for each significant mode based on the frequency (eigenvalue) of the mode. In essence the response contribution of each mode is determined for a series of time steps using a prescribed time-step interval, and the response at each time step is simply the superposition, or addition, of characteristic mode shapes adjusted by coefficients obtained from the integration procedure. Normally, only a few mode shapes are found to contribute significantly to the response, so that the modal superposition method produces a precise response with minimum computational effort.

c. Response spectrum analysis. In a response spectrum analysis, the step-by-step integration part of the dynamic analysis, described above for time-history analysis, is performed in the process of developing the response spectrum. The response spectrum may be envisioned as a display of the results of this part of the modal analysis, and it is presented in the form of "maximum" response versus frequency (or period). In the response spectrum modal analysis, eigenvalues, eigenvectors, and modal participation factors are computed and used in the analysis procedure just as they are in a time-history modal analysis. Precise "maximum" modal responses are easily calculated from a simple equation that relates these parameters and the appropriate spectral value that corresponds to the modal frequency.

d. Combining modal responses. The final step in a response spectrum analysis consists of correct superpositioning of the "maximum" modal responses; however, there is not a unique solution to this final step in the response spectrum method. This is because the exact mode contributions at the critical point in time when the response peaks are not available from a response spectrum representation of a particular ground motion. One advantage of a smooth design response spectrum is that it is a statistical representation, or an envelope, of the many possible ground motions that could occur at the site rather than only a single ground motion. The superposition of the maximum modal responses is accomplished by use of one of several statistical methods described in Chapter 7.

5-5. Types of Design Response Spectra

a. Probability level. Design response spectra are usually based statistically either on the mean, median (50th percentile probability level), or the median plus one standard deviation (84th percentile probability level), of the ground motion parameters for the records chosen. Design response spectra used for design of new RCC dams or for evaluation of the safety and serviceability of existing dams shall be based on the mean level of the ground motion parameters.

b. Type of spectrum required. Either a "site-specific" or a "standard" design response spectra shall be used to describe the design earthquakes. The type required shall be based on the seismic zone, the proximity of the seismic source, and the maximum height of the dam.

c. Site-specific design response spectra. The site-specific design response spectra should be developed based on earthquake source conditions, propagation path properties, and local foundation characteristics associated with the specific site. This type of design spectra may be established by anchoring a selected response spectral shape for the site to the estimated peak ground acceleration, or by estimating the design spectra directly using response spectral attenuation relationships, performing statistical analysis of strong-motion records, or applying theoretical (numerical) ground motion modeling. In the requirements that follow, a site is classified as a "high seismic risk site" when it is located within 20 kilometers of an active fault or area source in the western United States (WUS), or within a tectonic province in the eastern United States (EUS) where the source or province has a maximum local magnitude of 6.0 or greater. The boundary between the WUS and the EUS is defined as the eastern boundary of the Rocky Mountains. Site-specific design response spectra are required for:

(1) Dams greater than 100 feet in height located at a site classified as a "high seismic risk site."

(2) Dams greater than 100 feet in height located in Seismic Zone 2B, 3, or 4 even though the site is not classified as a "high seismic risk site."

(3) Dams not greater than 100 feet in height located in Seismic Zone 2B, 3, or 4 when the site is classified as a "high seismic risk site."

d. Standard design response spectra. Standard design response spectra are based on fixed spectral shapes established for very general site classifications such as rock or soil site. They ignore the effects of earthquake magnitude and distance, and the specific foundation characteristics at the site. The standard design spectra are usually "anchored" to the estimated peak ground acceleration (PGA) established for the design earthquake. The fixed spectral shape is usually presented such that it is normalized to a 1.0 g value of maximum ground acceleration. This normalized value can be easily checked by observing the spectral acceleration value from the spectrum plot for frequencies above about 50 cps where the response and the maximum ground acceleration coincide. Standard design response spectra are adapted to the severity of ground motion associated with the OBE or MCE by using the PGA as a scaling factor. The standard design response spectra can be used for:

(1) Dams greater than 100 feet in height located in Seismic Zone 0, 1, or 2A when the site is not classified as a "high seismic risk site."

(2) Dams not greater than 100 feet in height located in Seismic Zone 0, 1, or 2A.

(3) Dams not greater than 100 feet in height located in Seismic Zone 2B, 3, or 4 when the site is not classified as a "high seismic risk site."

e. Required design spectrum. When it is acceptable to use a standard design response spectrum to define the design earthquakes, the standard design spectrum shown in Figure 5-2 shall be used (Applied Technology Council 1984). This spectrum is considered conservative but reasonable for essential structures such as dams. It is fully described by only five control points on a tripartite plot. Table 5-1 presents the spectrum in equation format so it is easily developed for any damping value. The standard design spectrum shown in Figure 5-2 and defined in equation format in Table 5-1 is normalized to 1.0 g PGA. The standard spectrum shall be anchored to the PGA for the OBE and the MCE by using the appropriate scaling factors provided in Table 5-2. The correct scaling factors are selected based on the seismic zone location of the site using the seismic zone map shown in Figure 5-1.

5-6. Horizontal and Vertical Design Response Spectra

a. Site-specific design response spectra. When site-specific design response spectra are required in accordance with paragraph 5-5*c*, two independent design response spectra shall be developed, one to define the horizontal component of ground motion, and the second to define the vertical component. The vertical component of ground motion usually contains much higher frequency content than the horizontal component, therefore the spectral shape is quite different than that of the horizontal component. The PGA associated with the vertical component will also be different than the PGA of the horizontal component. Both values of PGA are dependent on the distance from the source, but for short distances, the PGA of the vertical component may actually exceed the PGA of the horizontal component.

b. Standard design response spectra. When it is acceptable to use standard design response spectra to define the design earthquakes, the horizontal component of ground motion shall be defined by anchoring the standard design response spectra for the appropriate damping factor developed from Table 5-1 with the scaling factor provided in Table 5-2. The vertical component of ground motion shall utilize the same standard design response spectrum used for the horizontal component, but it shall be scaled using the appropriate ratio of the PGA for the vertical component to the PGA for the horizontal component as provided in Figure 5-3. This ratio is based on the site to source distance (R) and the fundamental natural period of vibration of the structure.

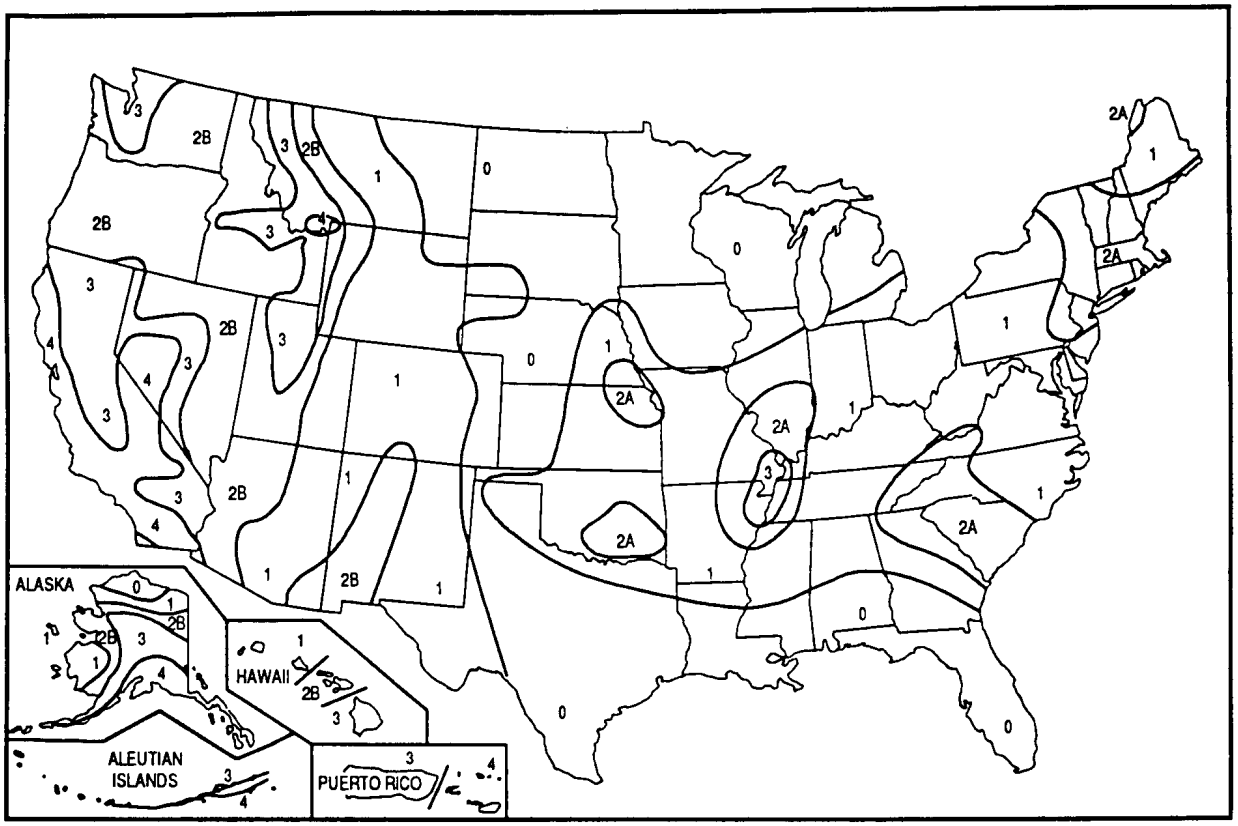
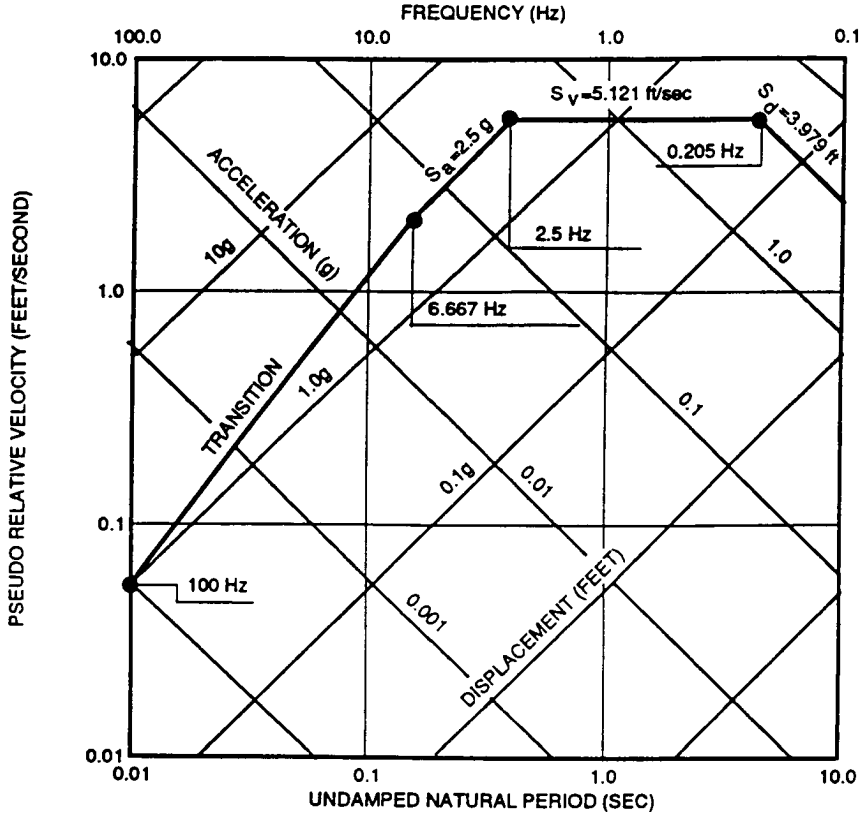
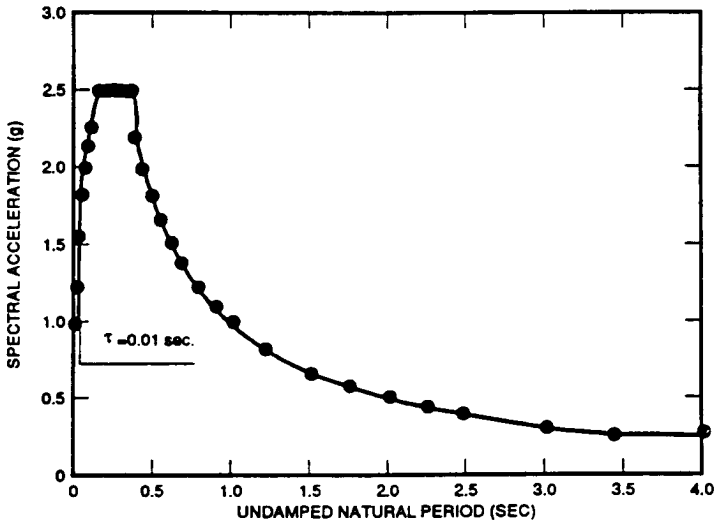


Figure 5-1. Seismic zone map of the United States. (Uniform Building Code, 1988 Edition)



Tripartite Logarithmic Representation (5% damped)



Arithmetic Plot of Spectral Acceleration versus Period (5% damped)

Figure 5-2. Standard design response spectra for horizontal component of ground motion - normalized to PGA = 1.0 g. (Applied Technology Council ATC-3-06 Tentative Provisions, 1984)

Table 5-1
Determining the Standard Design Response Spectrum for Horizontal Component of Ground Motion - Normalized to PGA = 1.0 g, for Any Value of β (Percent of Critical Damping)

T (sec)	f (Hz)	$S_{a(5\%)}$(g's)	K_1	
* 0.002	500.000	1.0000	0.00000	EQUATION 1
0.005	200.000	1.0000	0.00000	
0.008	125.000	1.0000	0.00000	
* 0.010	100.000	1.0000	0.00000	
0.020	50.000	1.2643	0.25596	
0.040	25.000	1.5985	0.51192	
0.060	16.667	1.8335	0.66164	
0.080	12.500	2.0210	0.76787	
0.100	10.000	2.1795	0.85028	
0.120	8.333	2.3182	0.97160	
* 0.150	6.667	2.5000	1.00000	
0.200	5.000	2.5000	1.00000	
0.250	4.000	2.5000	1.00000	
0.300	3.333	2.5000	1.00000	
0.350	2.857	2.5000	1.00000	
* 0.400	2.500	2.5000	1.00000	
0.450	2.222	2.2222		EQUATION 2
0.500	2.000	2.0000		
0.550	1.818	1.8182		
0.600	1.667	1.6667		
0.650	1.538	1.5385		
0.700	1.429	1.4286		
0.800	1.250	1.2500		
0.900	1.111	1.1111		
1.000	1.000	1.0000		
1.250	0.800	0.8000		
1.500	0.667	0.6667		
1.750	0.571	0.5714		
2.000	0.500	0.5000		
2.250	0.444	0.4444		
2.500	0.400	0.4000		
3.000	0.333	0.3333		
3.500	0.286	0.2857		
4.000	0.250	0.2500		
* 4.882	0.205	0.2048		
5.000	0.200	0.1953		
6.000	0.167	0.1356		
7.000	0.143	0.0996		
8.000	0.125	0.0763		
9.000	0.111	0.0603		
* 10.000	0.100	0.0488		

* INDICATES THE ONLY POINTS NEEDED TO SPECIFY THE RESPONSE SPECTRUM FOR COMPUTER PROGRAMS WITH LOGARITHMIC INTERPOLATION CAPABILITY.

DEFINITION OF TERMS

S_a = SPECTRAL ACCELERATION IN g's FOR β PERCENT OF CRITICAL DAMPING

$S_{a(5\%)}$ = SPECTRAL ACCELERATION IN g's FOR 5 PERCENT OF CRITICAL DAMPING

T = UNDAMPED NATURAL PERIOD, SECONDS

f = FREQUENCY, Hz

β = PERCENT OF CRITICAL DAMPING

K_1, K_2, K_3 = CORRECTION FACTORS USED TO DEVELOP A DESIGN RESPONSE SPECTRUM FOR β PERCENT OF CRITICAL DAMPING

CORRECTION FACTORS

K_1 = FACTOR SHOWN IN TABLE FOR VALUES OF THE NATURAL PERIOD T BETWEEN 0.000 AND 0.400

$K_2 = 1.466 - 0.2895 \, LN(\beta)$

$K_3 = LOG(2.5 \times K_2)$

EQUATIONS FOR S_a

WHEN $T \leq 0.400$ $S_a = 10.0^{K_1 K_3}$ (EQUATION 1)
$T > 0.400$ $S_a = K_2 S_{a(5\%)}$ (EQUATION 2)

NOTE: THE VALUES OF SPECTRAL VELOCITY S_v AND SPECTRAL DISPLACEMENT S_d CAN BE CALCULATED ONCE S_a IS KNOWN:

$S_v = 5.1207 \, (S_a T)$
$S_d = 0.81498 \, (S_a T^2)$

Table 5-2
Peak Ground Accelerations (PGA's) for Use in Scaling the Standard Design Response Spectra

Seismic Zone	PGA	
	Operating Basis Earthquake (OBE)	Maximum Credible Earthquake (MCE)
0	0.030	0.130
1	0.050	0.210
2A	0.095	0.360
2B	0.115	0.430
3	0.210	0.550
4	0.270	0.610

NOTES:

1. Refer to Figure 5-1 for the seismic zone maps.
2. PGA's are expressed as the decimal ratio of the acceleration due to gravity (g).
3. PGA's are obtained from curves of "Annual Risk of Exceedance vs. PGA" in Figure C1-7 of ATC-3 Tentative Provisions, April 1984.
4. The PGA for the OBE is based on a 50 percent chance of exceedance in 100 years.
5. The MCE is considered to be the event with a 5,000-year return period (annual risk of exceedance = 0.0002 chance/year).

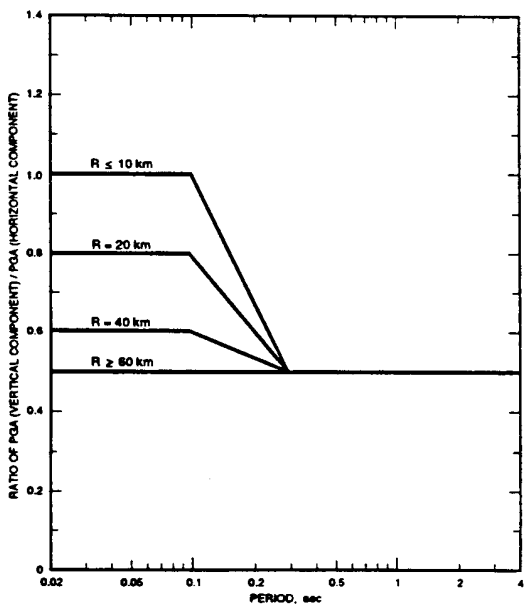


Figure 5-3. Ratio of PGA for the horizontal component to the PGA for the vertical component as a function of source to site distance (R) and the fundamental period of vibration of the structure