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2. File this change sheet in front of the publication for reference purposes.

FOR THE COMMANDER:

WILLIAM D. BROWN
Colonel, Corps of Engineers
Chief of Staff
1. **Purpose.** This manual presents the fundamental design principles and guidance concerning seepage considerations for design of new dams and the evaluation of existing projects.

2. **Applicability.** This manual is applicable to all HQUSACE/OCE elements and field operating activities having responsibility for the design and construction of civil works projects.

3. **Discussion.** All earth and rock-fill dams are subject to seepage through the embankment, foundation, and abutments. Concrete gravity and arch dams are subject to seepage through the foundation and abutments. Seepage control is necessary to prevent excessive uplift pressures, sloughing of the downstream slope, piping through the embankment and foundation, and erosion of material by loss into open joints in the foundation and abutments. The purpose of the project, i.e., long-term storage, flood control, etc., may impose limitations on the allowable quantity of seepage.

FOR THE COMMANDER:

ARThUR E. WILLIAMS  
Colonel, Corps of Engineers  
Chief of Staff

This manual supersedes EM 1110-2-1901 dated February 1952
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# Seepage Principles

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CHAPTER 1
INTRODUCTION

1-1. **Purpose.** This manual provides guidance and information concerning seepage analysis and control for dams.

1-2. **Applicability.** The provisions of this manual are applicable to all HQUSACE/OCE elements and field operating activities (FOA) having responsibility for seepage analysis and control for dams.

1-3 **References.** Appendix A contains a list of Government and non-Government references pertaining to this manual. Each reference is identified in the text by either the designated publication number or by author and date. Reference to cited material in tables and figures is identified throughout the manual by superscripted numbers (item 1, 2, etc.) that correspond to similarly numbered items in Appendix A.

1-4. **Objective and Scope.** The objective of this manual is to provide a guide for seepage analysis and control for dams.

1-5. **General Considerations.** All earth and rock-fill dams are subject to seepage through the embankment, foundation, and abutments. Concrete gravity and arch dams are subject to seepage through the foundation and abutments. Seepage control is necessary to prevent excessive uplift pressures, sloughing of the downstream slope, piping through the embankment and foundation, and erosion of material by loss into open joints in the foundation and abutments. The purpose of the project, i.e., long-term storage, flood control, etc., may impose limitations on the allowable quantity of seepage (Sowers 1977).
CHAPTER 2
DETERMINATION OF PERMEABILITY OF SOIL AND
CHEMICAL COMPOSITION OF WATER

2-1. Darcy's Law.

a. Development of Darcy's Law. Henry Darcy, a French engineer, conducted a laboratory experiment to study the flow of water in verticals and filters which he published in his 1856 treatise. The results of his experiment indicated that (Rouse and Ince 1957)

\[ v = ki \] (2-1)

or since \( Q = vA \)

\[ Q = kiAt \] (2-2)

or using \( q = \frac{Q}{t} \)

\[ q = kiA \] (2-3)

where

- \( v \) = discharge velocity
- \( k \) = Darcy's coefficient of permeability
- \( i \) = hydraulic gradient (head loss/length over which head loss occurs)
- \( Q \) = quantity of discharge
- \( A \) = cross-sectional area of flow
- \( t \) = time of flow
- \( q \) = rate of discharge

b. Extension to Inclined Soil Column. Darcy's law may be extended to flow through an inclined soil column given in figure 2-1 (Harr 1962). As indicated in equation 2-1, flow is a consequence of differences in total

---

(1) Commonly called the coefficient of permeability or the permeability.
head\(^1\) and not of pressure gradients (Harr 1962 and Bear 1972). As shown in figure 2-1, flow is directed from point A to point B even though the pressure at point B is greater than that at point A.

Figure 2-1. Darcy's law for flow through inclined soil column (prepared by WES)

\begin{center}
\begin{tabular}{c|c c c c c c c c}
\hline
POINT & \thead{} & \thead{} & \thead{} & \thead{} & \thead{} & \thead{} & \thead{} & \thead{} \\
\hline
A & \(h_A\) & \(P_A/\gamma_w\) & \(Z_A\) & \\
B & \(h_B\) & \(P_B/\gamma_w\) & \(Z_B\) & \\
\hline
\end{tabular}
\end{center}

\(^1\) The elevation head at any point is the distance from some arbitrary datum. The pressure head is the water pressure divided by the unit weight of the water. The total head is the sum of the elevation head and the pressure head.
c. Discharge Velocity and Seepage Velocity. The discharge velocity is defined as the quantity of fluid that flows through a unit cross-sectional area of the soil oriented at a right angle to the direction of flow in a unit time. The discharge velocity is used in determining the quantity of flow or rate of discharge through a soil. As flow can occur only through the interconnected pores of the soil, as shown in figure 2-2, the actual rate of

\[ v = \frac{Q}{A} \]

(2-4)
where

\[ \bar{v} = \text{seepage velocity} \]

\[ n = \text{porosity (ratio of volume of voids to the total volume of the soil mass)} \]

It follows that

\[ k = \frac{\bar{v} n}{1} \]  

Equation 2-5 is a useful expression in estimating field permeabilities using dye tracers (Soil Conservation Service 1978).

2-2. Range of Validity of Darcy's Law.

a. Lower Bound. Darcy's law (equations 2-1 through 2-3) applies to linear flow (adjacent flow lines are locally straight and parallel). For flows through soils, there are two situations where the validity of this linear relationship may not hold. For highly plastic clays of low permeability, there may be a threshold hydraulic gradient below which flow does not take place. Such conditions may occur in deeply buried clays and clay shales. For many practical seepage problems the rate of flow through these soil layers is so small that they can be considered to be impervious (Mitchell 1976, Chugaev 1971, Basak and Madhav 1979, and Muskat 1946).

b. Upper Bound. Of greater practical importance is the upper limit on the range of validity of Darcy's law. It has been recognized that, at very high flow rates, Darcy's law does not hold (Chugaev 1971). The upper limit is usually identified using Reynolds number, a dimensionless number that expresses the ratio of internal to viscous forces during flow. It is often used in fluid mechanics to distinguish between laminar flow (fluid layer flows alongside of another at approximately the same velocity with no macroscopic mixing of fluid particles) at low velocities and turbulent flow (velocity fluctuations, both parallel and transverse, are imposed upon the mean motion with mixing of the fluid particles) at high velocities. The Reynolds number for flow through soils is

\[ R = \frac{vD \rho}{\mu} \]

where

\[ R = \text{Reynolds number} \]

\[ D = \text{average diameter of soil particles} \]
\( \rho \) = density of fluid

\( \mu \) = coefficient of dynamic viscosity of fluid

The critical value of Reynolds number at which the flow in soils changes from laminar to turbulent has been determined experimentally by various investigators to range from 1 to 12 (Harr 1962 and Chugaev 1971). Assuming a water temperature of 20° C, substituting \( \rho = 998.2 \text{ kg/m}^3 \) and \( \mu = 1.002 \times 10^{-3} \text{ kg/m} \text{ sec} \) into equation 2-6, and assuming values of \( D \) and solving for \( v \) with \( \Re = 1 \) and \( \Re = 12 \) gives the relationship shown in figure 2-3 which defines the upper bound of the validity of Darcy's law. Depending on the discharge velocity, Darcy's law is generally applicable for silts through medium sands.

c. Turbulent Flow.

(1) Estimating Permeability from Empirical Equation. For flow through soils more pervious than medium sands, flow is likely to be turbulent. Under turbulent conditions, the seepage velocity in a material with monosized soil particles (coarse sands and/or gravels) can be estimated from the following equation (Wilkins 1956, Leps 1973, and Stephenson 1979).

\[
\bar{v} = wM^{0.5}i^{0.54}
\]

where

\( \bar{v} \) = seepage velocity in inches per second

\( w \) = an empirical constant, which depends on the shape and roughness of the soil particles and viscosity of water and varies from 33 for crushed gravel to 46 for polished marbles, in inch\(^{1/2}\) per second

\( M \) = hydraulic mean radius of the rock voids (for a given volume of particles equal to the volume of voids divided by the total surface area of the particles, or the void ratio divided by the surface area per unit volume of solids) in inches

\( i \) = hydraulic gradient

The coefficient of permeability is obtained from the seepage velocity using equation 2-5. For well-graded soils, the \( D_{50} \) size (50 percent finer by weight) can be used to calculate the hydraulic mean radius provided that the minus 1-in.-size material is less than 30 percent by weight. If there is more than 30 percent of minus 1-in.-size material, the permeability should be determined experimentally (Leps 1973).

(2) Determining Permeability Experimentally. Alternatively, for flow through soils more pervious than medium sands, the relationship between hydraulic gradient and discharge velocity can be determined experimentally (Cedergren 1977).
Figure 2-3. Boundary between laminar and turbulent flow determined using Reynolds number for temperature of 20° C (prepared by WES)

2-3. Coefficient of Permeability.

a. Darcy's (Engineer's) Coefficient of Permeability. The coefficient of permeability used in seepage analysis for dams is called the Darcy's or engineer's coefficient and is given by (Cedergren 1977)
or since \( Q = vA \)

\[
k = \frac{Q}{IAt}
\]

(2-9)

or using \( q = \frac{Q}{t} \)

\[
k = \frac{q}{IA}
\]

(2-10)

The coefficient of permeability is defined as the rate of discharge of water at a temperature of 20\(^\circ\) C under conditions of laminar flow through a unit cross-sectional area of a saturated soil medium. The coefficient of permeability has the dimensions of a velocity and is usually expressed in centimeters per second. Permeability computed on the basis of Darcy's law is limited to the conditions of laminar flow and complete saturation of the soil. Under conditions of partial saturation, the flow is in a transient state and is time dependent. To analyze natural flow conditions which depart from the Darcy flow condition, it is sometimes necessary to apply Darcy's law in conditions where it is not strictly valid. When this is done, the effects of turbulent flow and partial saturation on the permeability must be recognized and taken into consideration (Cedergren 1975).

**b. Intrinsic (Specific) Permeability.** The coefficient of permeability of a soil material varies for different pore fluids depending upon their density and viscosity as follows:

\[
k = k_o \frac{\gamma}{\mu}
\]

(2-11)

where

\[
k_o = \text{intrinsic permeability}
\]

\[
\gamma = \text{unit weight of pore fluid}
\]

\[
\mu = \text{viscosity of pore fluid}
\]

The intrinsic permeability has the dimensions of length squared and is expressed in square centimeters or Darcy's (equal to 1.01 x 10\(^{-8}\) cm\(^2\)). Figure 2-4 is a chart for the conversion of permeability values from one set of
Figure 2-4. Permeability conversion chart (courtesy of John Wiley & Sons, Inc.)

<table>
<thead>
<tr>
<th>cm/sec</th>
<th>1</th>
<th>10^-1</th>
<th>10^-2</th>
<th>10^-3</th>
<th>10^-4</th>
<th>10^-5</th>
<th>10^-6</th>
<th>10^-7</th>
<th>10^-8</th>
<th>10^-9</th>
<th>10^-10</th>
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<tr>
<td>m/min</td>
<td>10^-1</td>
<td>10^-2</td>
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<td>10^-4</td>
<td>10^-5</td>
<td>10^-6</td>
<td>10^-7</td>
<td>10^-8</td>
<td>10^-9</td>
<td>10^-10</td>
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<tr>
<td>μ/sec</td>
<td>1</td>
<td>10^-1</td>
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<td>10^-8</td>
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<td>10^-10</td>
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<tr>
<td>ft/sec</td>
<td>10^2</td>
<td>10^3</td>
<td>1</td>
<td>10^-1</td>
<td>10^-2</td>
<td>10^-3</td>
<td>10^-4</td>
<td>10^-5</td>
<td>10^-6</td>
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<td>10^-3</td>
<td>10^-4</td>
<td>10^-5</td>
<td>10^-6</td>
<td>10^-7</td>
</tr>
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- To convert from cm/sec to:
  - m/min: Multiply by 0.600
  - μ/sec: Multiply by 10^4
  - ft/sec: Multiply by 0.0328
  - ft/min: Multiply by 1.968
  - ft/yr: Multiply by 1.034 x 10^6
  - cm² (for water at 20°C): Multiply by 1.045 x 10^-3
  - Darcy (for water at 20°C): Multiply by 1.045 x 10^-3

For water at 20°C

*For water at 20°C
units to another (Lohman et al. 1972). Substituting equation 2-11 into equation 2-1

\[ \frac{v}{\mu} = k \frac{Y}{\mu} \]  

(2-12)

Equation 2-12 may be used when dealing with more than one fluid or with temperature variations. This is widely used in the petroleum industry where the presence of gas, oil, and water occur in multiphase flow systems (Freeze and Cherry 1979, and Bureau of Reclamation 1977). In seepage analysis for earth dams where we are primarily interested in the flow of water subject to small changes in temperature, this refinement is seldom required.

c. Transmissivity Factor. In order to describe the flow characteristics of an aquifer (saturated permeable geologic unit that can transmit significant quantities of water under ordinary hydraulic gradients), C. V. Thesis introduced the term transmissivity which is defined as (Bureau of Reclamation 1977)

\[ T = k t \]  

(2-13)

where

- \( T \) = transmissivity factor
- \( k \) = average permeability
- \( t \) = aquifer thickness

Transmissivity represents the rate of discharge for a gradient of unity through a vertical strip of aquifer one unit wide and has dimensions of length squared per unit time and is usually expressed in square feet per day.

2-4. Factors Influencing Permeability.

a. Range of Values of Permeability. No other property of soil exhibits a wider range of values (up to ten orders of magnitude) or shows greater directional (anisotropy) and spatial variability in a given deposit as does the coefficient of permeability. The approximate range in coefficients of permeability for soils and rocks is shown in figure 2-5 (Milligan 1976). Within the range, extreme variations of permeability in situ are possible due to the degree of stratification or heterogeneity of the soil deposit.

b. Variation of In Situ Permeability. Natural soil deposits are generally stratified in structure. Water-deposited soils are laid down in horizontal layers and are often more permeable in the horizontal than vertical direction. Windblown sands and silts are generally more permeable vertically than horizontally due to the presence of continuous vertical root holes. An important example of stratification is openwork gravel which may occur in
ordinary gravel or soil and have tremendous influence on the watertightness of dam foundations and abutments as shown in figure 2-6 (Cedergren 1977). Figure 2-6a shows a soil profile surmised from several drill holes. The grain size analysis of soil samples taken at frequent intervals erroneously indicated that the deposit was composed of relatively uniform sandy gravels. Laboratory permeability tests on disturbed samples produced coefficients of permeability of about $1 \times 10^{-6}$ cm/sec. Using this value of permeability, the probable seepage loss beneath the proposed dam was estimated to be 3 cu ft/day, which is an insignificant quantity. However, the design engineer had observed many openwork streaks in which the fines fraction of the material was almost completely absent along the banks of the river and noted that the ground-water table was level for several hundred feet away from the river and fluctuated rapidly with changes in river stage. Field pumping tests were conducted which indicated somewhat variable permeabilities but none approaching the magnitude of openwork gravels. Based upon the available data, the dam was designed with a cutoff trench to bedrock. During the excavation of the cutoff trench, streaks of openwork gravel were found throughout the foundation. A revised seepage computation based on a permeability of 30 cm/sec indicated that without the cutoff trench, the theoretical underseepage would be about 1,000,000 cu ft/day. If openwork gravel or other important discontinuities in earth dam foundations remain undetected, serious problems from excessive seepage and hydrostatic pressures will develop. This example illustrates the potential serious effects of deviations between the design assumptions and the as-built dam (Cedergren 1977). Also, thin continuous seams of cohesive soil can drastically alter the vertical flow through what would otherwise be a highly permeable site.

Figure 2-5. Approximate range in coefficient of permeability of soils and rocks (from Milligan)
a. Soil profile surmised from drill holes with estimated quantity of seepage under dam equal to 3 cu ft/day

b. Soil profile revealed by cutoff trench with estimated quantity of seepage under dam equal to 1,000,000 cu ft/day (without cutoff trench)

Figure 2-6. Influence of openwork gravel on underseepage (courtesy of John Wiley & Sons, Inc. 155)

c. Properties of the Seepage Fluid. The properties of the seepage fluid which influence the permeability of soils are the temperature, density, viscosity, and chemical composition.

(1) As shown in table 2-1, for the range of temperatures ordinarily encountered in seepage analysis of dams (0° C to 40° C) the density of water is nearly constant (varies less than 1 percent).

Table 2-1. Properties of Water

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>Density kg/m³</th>
<th>Viscosity kg/m sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>999.8</td>
<td>1.787 x 10⁻³</td>
</tr>
<tr>
<td>5</td>
<td>999.9</td>
<td>1.519</td>
</tr>
<tr>
<td>10</td>
<td>999.7</td>
<td>1.307</td>
</tr>
<tr>
<td>15</td>
<td>999.1</td>
<td>1.139</td>
</tr>
<tr>
<td>20</td>
<td>998.2</td>
<td>1.002</td>
</tr>
<tr>
<td>25</td>
<td>997.0</td>
<td>0.890</td>
</tr>
<tr>
<td>30</td>
<td>995.6</td>
<td>0.798</td>
</tr>
<tr>
<td>35</td>
<td>994.0</td>
<td>0.719</td>
</tr>
<tr>
<td>40</td>
<td>992.2</td>
<td>0.653</td>
</tr>
</tbody>
</table>

*Prepared by WES.*
(2) The viscosity varies up to 63.5 percent over the range of temperatures ordinarily encountered in seepage analysis of dams. As indicated in equation 2-11, the permeability is inversely proportional to the viscosity of the water. As given in table 2-1, the viscosity of water decreases as temperature increases. Therefore, the coefficient of permeability of the soil increases as the temperature of the water increases. Permeability tests are run at the most convenient temperature and reported at 20°C.

(3) The total dissolved salts (TDS) present in the seepage water may influence the permeability of the soil, particularly for cohesive soils (Quirk and Schofield 1955 and Cary, Walter, and Harstad 1943). Available data indicate that cohesive soils may be two to three orders of magnitude more permeable to seepage water containing moderate amounts of dissolved salts (less than 300 parts per million by weight) than the distilled water (Carry, Walter, and Harstad 1943).

d. Degree of Saturation. The degree of saturation of a soil

\[ S = \frac{V_w}{V_v} \times 100 \text{ percent} \]  \hspace{1cm} (2-14)

where

\[ S = \text{degree of saturation} \]
\[ V_w = \text{volume of water} \]
\[ V_v = \text{volume of voids} \]

has an important influence on permeability. A decrease in the degree of saturation causes a decrease in the permeability as shown in figure 2-7 (Lambe 1951). When the degree of saturation is less than 85 percent, much of the air would be continuous throughout the soil voids and Darcy's law would not hold. When the degree of saturation is greater than 85 percent, most of air present in the soil is in the form of small occluded bubbles and Darcy's law will be approximately valid. The ratio of the permeability of the unsaturated sand to the saturated sand at the same void ratio is given as (Scott 1963 and Parker and Thornton 1976)

\[ \frac{k_{us}}{k} = 1 - m \left( 1 - \frac{S}{100} \right) \quad 100 \geq S \geq 80 \]  \hspace{1cm} (2-15)

where

\[ k_{us} = \text{unsaturated permeability} \]
\[ m = \text{constant with values between 2 (uniform grain size) and 4 (well-graded materials)} \]
Figure 2-7. Permeability versus degree of saturation for various sands (courtesy of John Wiley & Sons, Inc.)

e. Hydraulic Gradient. The hydraulic gradient

\[ i = \frac{H}{L} \]  

(2-16)

where

\( H \) = head loss

\( L \) = length over which head loss occurs
at which permeability is measured can have a significant influence on the coefficient of permeability computed from Darcy's law under certain conditions. The maximum hydraulic gradient for which laminar flow occurs for a particular soil at a given density may be determined in the laboratory by plotting the discharge velocity

\[ v = \frac{Q}{Af} \]  

(2-17)

versus the hydraulic gradient as shown in figure 2-8. A straight line relationship indicates laminar flow

\[ k = \frac{v}{h} \]  

(2-18)

while deviations from the straight line at high gradients indicate turbulent flow. Darcy's law for a fine sand, as shown in figure 2-8, is valid only for the hydraulic gradient less than 2 for the loose state and 4.5 for the dense state. For soils larger than a fine sand, Darcy's law is valid for progressively smaller hydraulic gradients (Burmister 1948 and Burmister 1955).

f. Particle Size. For cohesive soils, the permeability increases with increases in clay mineral size and increase in void ratio (ratio of the volume of voids to the volume of solid particles in the soil mass) as shown in figure 2-9 (Yong and Warkentin 1966). For cohesionless soils, the size and shape of the soil particles influence the permeability. Allan Hazan conducted tests on filter sands for use in waterworks and found that for uniform loose clean sands the permeability was given by (Taylor 1948)

\[ k = 100 \frac{D_{10}^2}{D_{10}} \]  

(2-19)

where

\[ k = \text{coefficient of permeability in cm per second} \]

\[ D_{10} = \text{particle size in cm at which 10 percent of the material is finer by weight (also known as Hazen's effective size)} \]

(1) As shown in table 2-2, the exchangeable cation present influences the permeability of clay minerals at constant void ratio (Scott 1963). The permeabilities are much smaller when the exchangeable cation is sodium which is one of the reasons why sodium montmorillonite is used to seal reservoirs.
Figure 2-8. Determination of maximum hydraulic gradient for which laminar flow occurs for a fine sand (courtesy of American Society for Testing and Materials).

Hazen's experiments were made on sands for which $0.1 \text{ mm} < D_{10} < 0.3 \text{ mm}$ and the uniformity coefficient, $C_u < 5$, where

$$C_u = \frac{D_{60}}{D_{10}} \quad \text{(2-20)}$$

where

$C_u = \text{uniformity coefficient}$

$D_{60} = \text{particle size at which 60 percent of the material is finer by weight}$

The coefficient 100 is an average of many values which ranged from 41 to 146, but most of the values were from 81 to 117. Equation 2-19 makes no allowance for variations in shape of the soil particles or void ratio.
a. Edge view sketch to show relative size and shape of clay particles (dimension not shown in length)

b. Permeability versus void ratio for various clay minerals

Figure 2-9. Influence of particle size and void ratio on permeability of clay minerals (courtesy of Macmillan)
Table 2-2. Coefficients of Permeability for Different Exchange Cations and Void Ratios for Two Clay Minerals

<table>
<thead>
<tr>
<th>Clay Mineral</th>
<th>Exchangeable Cation</th>
<th>Void Ratio</th>
<th>Coefficient of Permeability cm/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Montmorillonite</td>
<td>Na</td>
<td>15</td>
<td>$8 \times 10^{-8}$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td></td>
<td>$1.5 \times 10^{-8}$</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>11</td>
<td>$8 \times 10^{-9}$</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td></td>
<td>$1 \times 10^{-7}$</td>
</tr>
<tr>
<td></td>
<td>Ca</td>
<td>8</td>
<td>$2 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td>$1 \times 10^{-7}$</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>9</td>
<td>$1 \times 10^{-7}$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kaolinite</td>
<td>Na</td>
<td>1.6-0.5</td>
<td>$1.5 \times 10^{-6}$ to $8 \times 10^{-7}$</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>1.6-1.1</td>
<td>$3 \times 10^{-6}$ to $9 \times 10^{-7}$</td>
</tr>
<tr>
<td></td>
<td>Ca</td>
<td>1.6-1.3</td>
<td>$1 \times 10^{-5}$ to $1.5 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>1.4-1.0</td>
<td>$1 \times 10^{-5}$ to $1.5 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Permeabilities are obtained by falling-head test on samples in consolidation apparatus. Results indicate the following:

For montmorillonite at void ratio 8 the order of permeability in terms of the exchangeable ion present is

$$K < Na < H < Ca$$

for kaolinite at void ratio 1.5 the order is

$$Na < K < Ca < H$$

For compacted soils it is also observed that the permeability is much lower ($x 10^{-1}$ to $10^{-2}$) in soils compacted slightly wet of optimum than in soils compacted dry of optimum; it is thought that this occurs because of the parallel arrangement of clay platelets in the wetter material after compaction.

---

*Courtesy of Addison-Wesley Publishing Company, Inc.*
g. **Particle Shape and Surface Roughness.** Cohesionless soil particles have different particle shapes and surface roughness dependent on the distance they have been transported by flowing water from the place of original erosion. As shown in table 2-3, the measured permeability is several orders of magnitude lower for angular sand particles with rough surfaces than for rounded sand particles with smooth surfaces (Burmister 1948). For uniform cohesionless soils, crushing of particles during compaction with resulting decrease in permeability occurs to a higher degree in soils with angular shapes and rough surfaces than in soils with rounded shapes and smooth surfaces. Crushing of particles during compaction leads to an increase in the amount of silt-sized particles (smaller than No. 200 sieve or 0.074 mm) which results in lower permeability. For this and other reasons (cementation in limestones and arching due to particle angularity) crushed rock is generally not used for filters in earth dams. Also, table 2-3 compares the measured permeability with the permeability computed from equation 2-19 developed by Hazen for uniform loose clean sands. The agreement between measured and computed permeability is within one order of magnitude for uniform sands and glass spheres. Therefore, Hazen's equation should be used only for uniform sands (sphericity and roundness > 0.90). The sphericity and roundness may be estimated for sands using figure 2-10 (Krumbein and Sloss 1951).

h. **Void Ratio.** The permeability increases as the void ratio increases.

\[ e = \frac{V_v}{V_s} \]  

(2-21)

where

\[ e = \text{void ratio} \]

\[ V_v = \text{volume of solids} \]

There are considerable laboratory test data, shown in figure 2-11, which indicate that a plot of void ratio versus log of coefficient of permeability is frequently a straight line (Lambe and Whitman 1969).

i. **Amount and Type of Fines Present.** The permeability of sands and gravels varies significantly with the amount and type of fines (material smaller than the No. 200 sieve) (Barber and Sawyer 1952; Fenn 1966; Younger and Lim 1972; Strohm, Nettles, and Calhoun 1967; Nettles and Calhoun 1967, and Loudon 1952). As shown in figure 2-12a, the addition of 2.5 percent, by dry weight, silt fines to concrete sand results in an order of magnitude decrease in permeability (Barber and Sawyer 1952). The addition of 6.5 percent silt fines to concrete sand decreases the permeability two orders of magnitude. Similar results are obtained by the addition of somewhat larger amounts of clay and limestone fines to concrete sand. As shown in figure 2-12b, the addition of 2.0 percent silt fines to a sand-gravel mixture results in an order of magnitude decrease in permeability (Barber and Sawyer 1952). The addition of 4.2 percent silt fines to sand-gravel mixture decreases the permeability two orders of magnitude. Similar results are obtained by the addition of...
Table 2-3. Influence of Particle Shape and Surface Roughness on Permeability of Sand\(^{(1)}\)

<table>
<thead>
<tr>
<th>Type of Material</th>
<th>Surface Roughness</th>
<th>Sphericity</th>
<th>Roundness</th>
<th>$C_u$</th>
<th>$C_c$</th>
<th>Void Ratio</th>
<th>Relative Density percent</th>
<th>Coefficient of Permeability cm/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crushed Sudeten granite</td>
<td>Very rough</td>
<td>0.68</td>
<td>0.10</td>
<td>3.09</td>
<td>1.54</td>
<td>0.543</td>
<td>79</td>
<td>$5.00 \times 10^{-5}$</td>
</tr>
<tr>
<td>Nysd Klodzké River sand</td>
<td>Very rough</td>
<td>0.75</td>
<td>0.15</td>
<td>5.47</td>
<td>2.20</td>
<td>0.451</td>
<td>79</td>
<td>$5.19 \times 10^{-6}$</td>
</tr>
<tr>
<td>Odra River sand</td>
<td>Rough to smooth</td>
<td>0.84</td>
<td>0.55</td>
<td>2.58</td>
<td>0.87</td>
<td>0.595</td>
<td>58</td>
<td>$9.41 \times 10^{-4}$</td>
</tr>
<tr>
<td>Odra River sand</td>
<td>Rough to smooth</td>
<td>0.87</td>
<td>0.65</td>
<td>2.42</td>
<td>0.83</td>
<td>0.608</td>
<td>63</td>
<td>$5.44 \times 10^{-3}$</td>
</tr>
<tr>
<td>Baltic Beach sand</td>
<td>Smooth</td>
<td>0.87</td>
<td>0.85</td>
<td>1.91</td>
<td>0.89</td>
<td>0.720</td>
<td>38</td>
<td>$1.83 \times 10^{-3}$</td>
</tr>
<tr>
<td>Glass spheres</td>
<td>Very smooth</td>
<td>1.00</td>
<td>1.00</td>
<td>1.38</td>
<td>0.92</td>
<td>0.691</td>
<td>67</td>
<td>$6.45 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

\(^{(1)}\)Courtesy of British Geotechnical Society\(^{146}\).

\(^{(2)}\) \(C_u = \frac{D_{60}}{D_{10}}\).

\(^{(3)}\) \(C_c = \frac{D_{30}^2}{D_{10} \times D_{60}}\).

\(^{(4)}\) \(k = 100D_{10}\).
somewhat smaller amounts of clay and larger amounts of limestone, respectively, to a sand-gravel mixture. As shown in figure 2-12c, the addition of about 1 percent calcium montmorillonite fines to a uniform fine sand results in an order of magnitude decrease in permeability, while over 10 percent kaolinite fines would be required for a similar reduction in permeability (Fenn 1966).

j. Summary of Factors Influencing Permeability. The significant influence that various factors exert on the permeability emphasizes the importance of duplicating field conditions when determining permeability in the laboratory.

2-5. Indirect Methods for Determining Permeability.

a. Hazen's Equation. For uniform loose clean sands, classified SP in the Unified Soil Classification System (U. S. Army Engineer Waterways Experiment Station 1960), the permeability may be estimated from the previously given Hazen's equation (Taylor 1948)

\[ k = 100 \frac{D_{10}^2}{D_{10}} \]  

(2-22)

where \( k \) is in cm per second and \( D_{10} \) is in cm.
Figure 2-11. Relationship between void ratio and log of permeability (courtesy of Wiley)
a. Effect of fines on permeability of concrete sand (from Barber and Sawyer)  

b. Effect of fines on permeability of sand-gravel mixture (from Barber and Sawyer)

c. Effect of fines on permeability of uniform fine sand (from Fenn)

Figure 2-12. Influence of type and amount of fines on permeability of concrete sand, sand-gravel mixture, and uniform fine sand (prepared by WES)
b. Masch and Denny Method. For uniform or nonuniform dense clean sands, classified SP or SW in the Unified Soil Classification System (U. S. Army Engineer Waterways Experiment Station 1960), the permeability may be estimated from an empirical method developed by Masch and Denny 1966, and Denny 1965. The gradation curve is plotted in Krumbein 4 units (Krumbein and Pettijohn 1938) (using the chart in figure 2-13) as shown in figure 2-14 where

\[ \phi = -\log_2 d = - \frac{\log_{10} d}{\log_{10} 2} = -3.322 \log_{10} d \]  

(2-23)

where

- \( \phi \) = phi scale units used to describe grain size distribution
- \( d \) = grain size diameter in mm

The inclusive standard deviation is used as a measure of the spread of the gradation curve where (Masch and Denny 1966)

\[ \sigma_I = \frac{d_{16} - d_{84}}{4} + \frac{d_5 - d_{95}}{6.6} \]  

(2-24)

where

- \( \sigma_I \) = inclusive standard deviation
- \( d_{16} \) = grain size in \( \phi \) units at which 16 percent is finer
- \( d_{84} \) = grain size in \( \phi \) units at which 84 percent is finer
- \( d_5 \) = grain size in \( \phi \) units at which 5 percent is finer
- \( d_{95} \) = grain size in \( \phi \) units at which 95 percent is finer
Figure 2-13. Conversion chart for $\phi$ and grain size in mm, for the range 0.01 to 10 mm (prepared by WES)
a. Grain size distribution

b. Coefficient of permeability versus median grain size

Figure 2-14. Masch and Denny relationship for permeability as a function of median grain size and inclusive standard deviation (courtesy of Prentice-Hall)
The median grain size, $d_{50}$, in $\phi$ units, is determined from the gradation curve as shown in figure 2-14a. Then knowing $\sigma_I$ and $d_{50}$, the coefficient of permeability in cm per minute can be obtained from figure 2-14b (Freeze and Cherry 1979).

c. Kozeny-Carman Equation. For uniform loose to dense clean sands classified SP in the Unified Soil Classification System (U. S. Army Engineer Waterways Experiment Station 1960), the permeability may be estimated using the Kozeny-Carman equation (Loudon 1952 and Perloff and Baron 1976)

$$k = \frac{1}{C_s^2 T_o^2} \frac{\gamma_w}{\mu} \frac{e^3}{1 + e}$$  \hspace{1cm} (2-25)

where

$k = \text{coefficient of permeability}$

$\gamma_w = \text{unit weight of fluid}$

$e = \text{void ratio}$

$C_s = \text{shape factor corresponding to a particular flow channel}$

$T_o = \text{tortuosity factor related to the degree of sinuous flow}$

$s_s = \text{specific surface (surface area of solids/volume of solids)}$

$\mu = \text{coefficient of viscosity of fluid}$

For sands and silt-sized (finer than 0.074 mm and coarser than 0.005 mm) particles $C_s T_o^2 = 5$ is a good approximation (Perloff and Baron 1976). The specific surface may be obtained from (Loudon 1952)

$$s_s = A(X_1 s_1 + X_2 s_2 + \ldots + X_n s_n)$$  \hspace{1cm} (2-26)

where

$s_s = \text{specific surface}$

$A = \text{angularity factor}$

$X_i = \text{percentage of total soil sample between adjacent sieves expressed as a decimal}$

$s_1 = \text{specific surface of spheres uniformly distributed in size between the mesh sizes of adjacent sieves}$

2-26
The angularity factor, $A$, which varies from 1.0 for glass spheres to 1.8 for crushed glass, may be determined by microscopic examination of the soil or estimated from table 2-4 (Loudon 1952). The specific surface of spheres, $S_i$, between the mesh sizes $d_x$ and $d_y$ is (Loudon 1952):

$$S_i = \frac{6}{\sqrt{d_x d_y}}$$

Specific surfaces of spheres lying between selected U. S. standard sieves is given in table 2-5.

d. **Correlation of In Situ Horizontal Permeability and Hazen's Effective Grain Size.** For natural fine to medium, relatively uniform sands, classified SP or SW in the Unified Soil Classification System (U. S. Army Engineer Waterways Experiment Station 1960), in the middle and lower Mississippi River Valley, the in situ horizontal permeability may be estimated from the Hazen's effective size as shown in figure 2-15 (U. S. Army Engineer Waterways Experiment Station 1956a). The relationship given in figure 2-15 should not be used outside the geographic area for which it was developed. A similar relationship between transmissivity and median grain size of sands is available for the Arkansas River Valley (Bedinger 1961).

**Table 2-4. Angularity Factor for Soil Grains**

<table>
<thead>
<tr>
<th>Type of Material</th>
<th>Description</th>
<th>Angularity Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass sphere</td>
<td>Well rounded</td>
<td>1.0</td>
</tr>
<tr>
<td>Natural sand</td>
<td>Rounded</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>Subrounded</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>Subangular</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>Angular</td>
<td>1.4</td>
</tr>
<tr>
<td>Crushed rock</td>
<td>Quartzite</td>
<td>1.5</td>
</tr>
<tr>
<td>Crushed rock</td>
<td>Basalt</td>
<td>1.6</td>
</tr>
<tr>
<td>Crushed glass</td>
<td>Pyrex</td>
<td>1.8</td>
</tr>
</tbody>
</table>

(a) Courtesy of the Institution of Civil Engineering.
Table 2-5. Specific Surface of Spheres Lying Between Selected U. S. Standard Sieve Sizes

<table>
<thead>
<tr>
<th>U. S. Standard Sieve Numbers</th>
<th>Specific Surface (b) 1/cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 to 6</td>
<td>382</td>
</tr>
<tr>
<td>6 to 8</td>
<td>538</td>
</tr>
<tr>
<td>8 to 10</td>
<td>696</td>
</tr>
<tr>
<td>10 to 16</td>
<td>985</td>
</tr>
<tr>
<td>16 to 20</td>
<td>1524</td>
</tr>
<tr>
<td>20 to 30</td>
<td>2178</td>
</tr>
<tr>
<td>30 to 40</td>
<td>3080</td>
</tr>
<tr>
<td>40 to 50</td>
<td>4318</td>
</tr>
<tr>
<td>50 to 70</td>
<td>6089</td>
</tr>
<tr>
<td>70 to 100</td>
<td>8574</td>
</tr>
<tr>
<td>100 to 140</td>
<td>12199</td>
</tr>
<tr>
<td>140 to 200</td>
<td>17400</td>
</tr>
</tbody>
</table>

(a) Prepared by WES.

(b) \[ S_I = \frac{6}{\sqrt{d_x d_y}} \]


a. General. Laboratory tests described in EM 1110-2-1906 can be used to determine the coefficient of permeability of a soil. Unless otherwise required, the coefficient of permeability shall be determined using deaired distilled water and completely saturated soil specimens. The apparatus used
for permeability testing may vary depending upon whether the sample is fine-grained or coarse-grained, undisturbed, remolded, or compacted, and saturated or unsaturated. The permeability of remolded coarse-grained soils is determined in permeameter cylinders, while the permeability of undisturbed coarse-grained soils in a vertical direction can be determined using the sampling tube as a permeameter. Samples which have become segregated or contaminated with drilling mud during sampling operations will not give reliable results. The permeability of remolded coarse-grained soils is generally used to approximate the permeability of undisturbed coarse-grained soils in a horizontal direction. Usually the laboratory permeability of remolded coarse-grained soils is considerably less than the horizontal permeability of the coarse-grained soil in the field, so the approximation may not be conservative. Pressure cylinders and consolidometers are used for fine-grained soils.
in the remolded or undisturbed state. Fine-grained soils can be tested with the specimen oriented to obtain the permeability in either the vertical or horizontal direction.

b. Possible Errors. There are several possible errors in determining permeability in the laboratory (FM 1110-2-1906; Olson and Daniel 1979; and Mitchell, Guzikowski, and Villet 1978).

(1) Use of samples that are not representative of actual field conditions. This can be minimized by thorough field investigation, attention to details (take undisturbed samples from test fills for determination of permeability of embankment materials, sampling along faults, fissures, clay seams, and sand partings for determination of permeability of the dam foundation), and by the use of large samples.

(2) Orientation of the in situ stratum to the direction of seepage flow is seldom duplicated in the laboratory. This can be overcome by obtaining the permeability of the soil (embankment material and/or foundation) in both the vertical and horizontal direction.

(3) Incorrect hydraulic gradient used in the laboratory test. The hydraulic gradient used in the laboratory should cover the range of expected hydraulic gradient in situ. Where possible the hydraulic gradient should be selected so that the flow is laminar (straight line relationship between discharge versus hydraulic gradient) and Darcy's law will be applicable. It is usually not practical to achieve laminar flow for coarser soils, and the laboratory test should be run at the hydraulic gradient anticipated in the field.

(4) Air dissolved in the water. As water enters the specimen, small quantities of air dissolved in the water will tend to collect as fine bubbles at the soil-water interface and reduce the permeability with increasing time. Permeability tests on saturated specimens should show no significant decrease in permeability with time if properly deaired distilled water is used. However, if such a decrease in permeability occurs, then a prefilter, consisting of a layer of the same material as the test specimen, should be used between the deaired distilled water reservoir and the test specimen to remove the air remaining in solution.

(5) Leakage along the sides of the permeameter can result in an increased permeability. One major advantage of the triaxial compression chamber for permeability tests is that the specimen is confined by a flexible membrane which is pressed tightly against the specimen by the chamber pressure thus reducing the possibility for leakage along the sides.


a. Hydrologic Cycle. Precipitation, runoff, storage, and evaporation of the earth's water follow an unending sequence called the hydraulic cycle, as shown in figure 2-16. Radiation from the sun evaporates water from the oceans into the atmosphere. The moisture is condensed and rises to form cloud formations. From these clouds, the earth receives precipitation of rain,
snow, sleet, or hail which runs into lakes and streams or seeps into the soil and thence into the underlying rock formations. The percolating water moves through the saturated subsurface materials and may reappear at the surface, at a lower elevation than the level where it entered the ground, in the form of springs and seeps which maintain the flow of streams in dry periods (TM 5-545; Bureau of Reclamation 1977; and Johnson Division, Universal Oil Products 1972).

b. Water Table. The surface below which the soil or rock is saturated is the water table, as shown in figure 2-17. The water table is not a level surface but varies in shape and slope depending upon the variations in permeability and areas of recharge and discharge. In general, the water table reflects the surface topography but with less relief. Ground water is said to be perched if it is separated from the main water table by unsaturated materials, as shown in figure 2-17. An aquifer is a saturated permeable geologic unit that can transmit significant quantities of water under ordinary hydraulic gradients. An unconfined aquifer is one that does not have a confining layer overlying it as shown in figure 2-17. The water table, or upper surface of the saturated ground water is in direct contact with the atmosphere through the open pores of the overlying material and movement of the ground water is in direct response to gravity. The aquifer may be a layer of gravel or sand, permeable sedimentary rocks such as sandstones or limestones, a rubbly zone between lava flows, or even a large body of massive rock, such as fractured

Figure 2-16. Schematic diagram of the hydrologic cycle (courtesy of Johnson Division, Universal Oil Products 189)
Figure 2-17. Groundwater relationships (courtesy of Soil Conservation Service)
granite, which has sizable openings. An aquiclude is a saturated geologic unit that is incapable of transmitting significant quantities of water under ordinary hydraulic gradients as shown in figure 2-17. A confined or artesian aquifer has an overlying confining layer of lower permeability than the aquifer and has only an indirect or distant connection with the atmosphere as shown in figure 2-17. The water in the artesian aquifer is under pressure and if the aquifer is penetrated by a tightly cased well or a piezometer, the water will rise above the bottom of the confining layer to an elevation at which it is in balance with atmospheric pressure. If this elevation is greater than that of the land surface at the well, artesian water will flow from the well as shown in figure 2-17. The imaginary surface, conforming to the elevations to which water will rise in wells penetrating an artesian aquifer, is the piezometric surface as shown in figure 2-17 (Soil Conservation Service 1978, TM 5-545, Bureau of Reclamation 1977, Freeze and Cherry 1979, and Anonymous 1980).


a. General. In sands it is difficult to obtain undisturbed soil samples for laboratory testing and the structure (void ratio, stratification, etc.) has an important influence on permeability. Therefore, field tests for determining permeability are necessary. Because sampling operations do not necessarily indicate the relative perviousness of foundations containing large amounts of gravelly materials, field pumping tests are required to determine the foundation permeability for dams where positive measures are not proposed to completely cut off underseepage in the gravelly formations.

b. Test Pits and Bore Hole Tests.. In sands and gravels above the ground-water level, field tests are normally carried out by measuring the downward seepage from test pits or shallow boreholes (Cedergren 1977). Below the ground-water table information about the order of magnitude and variability of the coefficient of permeability may be obtained by conducting falling head permeability tests in the exploratory boring as drilling proceeds. The hole is cased from the ground surface to the top of the zone to be tested and extends without support for a suitable depth below the casing. If the pervious stratum is not too thick, the uncased hole is extended throughout the full thickness, otherwise the uncased hole penetrates only a part of the pervious stratum. Water is added to raise the water level in the casing and then the water level descends toward its equilibrium position. The elevation of the water level is measured as a function of time and the coefficient of permeability is calculated (Terzaghi and Peck 1967).

\[
k = \frac{1}{C} \frac{\Delta h}{A \Delta t / \Delta \phi / m}
\]

where

\[
k = \text{coefficient of permeability}
\]
A = inside cross-sectional area of casing

$\Delta h$ = drop in water level in casing during time interval $\Delta t$

$\Delta t$ = time interval

C = dimensionless quantity depending on shape of cylindrical hole and depth of penetration into pervious zone (see figure 2-18)

$r'_o$ = mean radius of hole below casing

$h'_m$ = mean distance during time interval $\Delta t$ from water level in casing to equilibrium water level in pervious zone

The falling head field permeability test often gives an observed permeability that is too low because silt particles which are suspended in the water may form a filter skin over the walls and bottom of the hole in the pervious material. The results of such tests are little more than an indication of the order of magnitude of the in situ permeability. More reliable data are obtained from field pumping tests.

c. Field Pumping Tests. The most reliable method for determining in situ permeability is a field pumping test on a test well which fully penetrates the aquifer. The test procedures for equilibrium (steady-state flow) and nonequilibrium (transient flow) are given in Appendix III to TM 5-818-5 and Civil Works Engineer Letter 63-16 (U. S. Army Corps of Engineers 1963). The ratio of the horizontal to vertical permeability can be determined from specially conducted field pumping tests (Mansur and Dietrich 1965).


a. Ground Water. The chemical composition of the ground water is important because some ground waters are highly corrosive to metal screens, pipes, and pumps, or may contain dissolved minerals or carbonates which form incrustations in wells or filters and, with time, cause clogging and reduced efficiency of the dewatering or drainage system. Indications of corrosive and incrusting waters are given in table 2-6 (TM 5-818-5; and Johnson Division, Universal Oil Products 1972). General information concerning ground-water properties is available in an Atlas (Pettyjohn et al. 1979). Sampling, sample preservation, and chemical analysis of ground water is covered in handbooks (Moser and Huibregtse 1976, and Environmental Protection Agency 1976).

b. River (or Reservoir) Water. The total amount of cations (calcium, magnesium, potassium, and sodium) in the river water (for dams not yet constructed) and in the reservoir water (for existing dams) significantly influences the erosion through a possible crack in the core of the dam (Perry 1975). Usually, as the total amount of cations in the eroding water decreases, the erodibility of the soil increases. For dams constructed of dispersive clay, the susceptibility of the dam to piping depends, in part, upon the total amount of cations in the seepage water (Perry 1979).
Figure 2-18. Field permeability test in bore hole (courtesy of John Wiley and Sons175)
Table 2-6. Indicators of Corrosive and Incrusting Waters\(^{(a)}\)

<table>
<thead>
<tr>
<th>Indicators of Corrosive Water</th>
<th>Indicators of Incrusting Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. A pH less than 7</td>
<td>1. a pH greater than 7</td>
</tr>
<tr>
<td>2. Dissolved oxygen in excess of 2 ppm(^{(b)})</td>
<td>2. Total iron (Fe) in excess of 2 ppm</td>
</tr>
<tr>
<td>3. Hydrogen sulfide ((H_2S)) in excess of 1 ppm, detected by a rotten egg odor</td>
<td>3. Total manganese ((Mn)) in excess of 1 ppm in conjunction with a high pH and the presence of oxygen</td>
</tr>
<tr>
<td>4. Total dissolved solids in excess of 1000 ppm indicates an ability to conduct electric current great enough to cause serious electrolytic corrosion</td>
<td>4. Total carbonate hardness in excess of 300 ppm</td>
</tr>
<tr>
<td>5. Carbon dioxide ((CO_2)) in excess of 50 ppm</td>
<td></td>
</tr>
<tr>
<td>6. Chlorides ((Cl)) in excess of 500 ppm</td>
<td></td>
</tr>
</tbody>
</table>

\(^{(a)}\) From TM 5-818-51.

\(^{(b)}\) ppm = parts per million.
CHAPTER 3
DETERMINATION OF PERMEABILITY OF ROCK

3-1. Permeabilities of Rock Masses. Permeability of rock, as with soil, is a measure of the ease with which fluids may travel through a medium under the influence of a driving force. The term "permeability," however, has several definitions for describing the flow of water in rock masses.

a. Coefficient of Permeability. The engineer's coefficient, or Darcy's coefficient, is normally referred to as simply the "coefficient of permeability." It is defined as the discharge velocity through a unit area under a unit hydraulic gradient and is dependent upon the properties of the medium, as well as the viscosity and density of the fluid (Section 2-3.a.).

b. Intrinsic Permeability. The physicist's coefficient, or the intrinsic permeability, is occasionally used in the determination of the hydraulic conductivity of a rock mass. It is defined as the volume of a fluid of unit viscosity passing through a unit cross section of a medium in unit time under the action of a unit pressure gradient (Section 2-3.b.). The intrinsic permeability thus varies with the porosity of the medium and is independent of both the viscosity and the density of the fluid.

c. Equivalent Permeability. The complex system of interconnected void space in a rock mass may be described in terms of an equivalent porous continuum, and the flow assumed to occur uniformly throughout the mass rather than within individual passageways. Under these assumptions the term equivalent permeability is used to describe the permeability of a rock mass.

d. Parallel Plate Permeability. The permeability of a fissure or a fissure set is occasionally determined by modeling the rock mass conditions with parallel plates. The parallel plate permeability can be computed from the value of the aperture between the plates (Ziegler 1976) and this permeability provides inferences for use in the modeled rock mass. The accuracy of computed parallel plate permeabilities has been verified consistently with laboratory tests (Snow 1965).

e. Fissure Permeability. Fissure permeability is the permeability of an individual fissure or a set of fissures and, whether measured in the laboratory or in the field, is determined using the parallel plate permeability theory. Fissure permeability is determined using an equivalent parallel plate aperture, rather than the actual fissure aperture. This in effect incorporates roughness into the parallel plate law.

3-2. Flow Characteristics in Rock Masses. The determination of the permeability of a rock mass, whether it be a rock slope, dam foundation, or dam abutment, can only be accomplished after certain criteria are defined or specified.

a. Continuum Approximations. From an overall regional point of view, most rock masses may be treated as continua, for all practical purposes. One of the primary considerations in selecting the continuum approach, for evaluating the permeability of a rock mass, is the relative size, frequency, and
orientation of the inherent discontinuities in the rock mass, compared with the size of the area of interest or area under study. If the flow characteristics of a rock mass are to be treated as those of continua, the net variation in flow over the study area should be relatively small, and the frequency and orientation of the discontinuities should be such that they provide an overall averaging effect on the flow with respect to the area of interest.

b. Discontinuum Approximations. As areas of investigation become smaller and smaller, i.e. more specific, the inherent discontinuities in rock masses start to play larger and larger roles in the interpretation of groundwater flow and its path of movement. The permeabilities or relative permeabilities of individual fissures and fissure systems are important for estimating the amount of seepage into various sections of underground excavations such as powerhouse tunnels or diversions, or through specific strata in a dam foundation or abutment. Discontinuum approximations of permeability are normally used when the flow in the area of investigation is governed either by a single fissure or by a fissure system.

(1) Single Fissure Flow. The permeability of single fissures can be very important in karstic terrains, basalt, or rhyolite flows, or in areas where tunnels are driven through faulted zones. The flow through a single fissure, under certain geologic conditions, can be the key to estimating the seepage through a dam foundation or abutment, or for determining the best route for a proposed tunnel.

(2) Flow Through Fissure Systems. Fissure systems such as joints, fractures, and bedding planes can yield, or contribute to, unpredictable flow paths, seepage patterns, and uplift pressures. The permeability of fissure systems should be evaluated by a discontinuum method of analysis when the size, frequency, and orientation of such systems make a continuum approach to the area under investigation unrealistic.

c. Ground-water Velocity. Flow of water in rock masses is generally considered to be governed by one of two laws. Under conditions of laminar flow, Darcy's law, previously presented as equation 2-1 in Chapter 2 is assumed to govern the flow. For turbulent flow conditions in rock there is a nonlinear flow velocity versus hydraulic gradient relationship and equations presented by Forchheimer (1914) and Missbach (1937) are assumed to govern the flow. The Missbach law is the most convenient to analyze and it takes either of two forms,

\[ v^m = k'i \]  \hspace{1cm} (3-1)

or

\[ v = B'i^a \]  \hspace{1cm} (3-2)
where

\[ m \quad \text{and} \quad \alpha = \text{degrees of nonlinearity} \]

\[ k' \quad \text{and} \quad B' = \text{turbulent coefficients of permeability} \]

Since the two equations are identical, resolving them indicates that \( m = \frac{1}{\alpha} \)
and \( k' = B' \frac{1}{\alpha} \). The use of the above laws, however, is usually restricted
to a homogeneous, isotropic, porous continuum. Since in soil and rock masses
there are complex systems of interconnected void spaces, an equivalent rather
than an absolute permeability should be determined. The coefficient of equiv-
alent permeability from the continuum approach assumes that flow occurs uni-
formly throughout the mass rather than within individual passageways. There-
fore, for equivalent permeability, Darcy's law and Missbach's law are written, respectively,

\[
v = k_e \quad (3-3)
\]

and

\[
v^m = k'_e \quad (3-4)
\]

where

\[ k_e = \text{laminar equivalent permeability} \]

\[ k'_e = \text{turbulent equivalent permeability} \]

The continuum approach, in some cases, is not applicable and therefore, the
discontinuum method of analysis for evaluating an equivalent permeability
should be used. Formulae for the discontinuum analysis for equivalent
permeability are presented later with various other methods of analysis.

been developed for determining or estimating rock mass permeabilities. All of
the available testing, as well as analytical techniques should be considered
and evaluated for each individual study, in order to optimize the advantages
and minimize the disadvantages inherent within each method for determining the
permeability of a rock mass.

a. Laboratory Permeability Tests. Laboratory permeability tests are
used for evaluating the permeability of rock cores or samples, determining the
flow characteristics of rock fissures, and performing parametric studies of
the factors affecting the permeability of rock masses. Laboratory test
methods for permeability provide a convenient research and evaluation tool
because a variety of parameters may be controlled and varied to yield a broad spectrum of conditions which may be encountered in a rock mass.

(1) Model Tests. Model tests are conducted by constructing parallel plate models to simulate given geologic information. The tests generally tend to be a parametric evaluation, but have been used extensively in evaluating both theoretical and empirical rock mass permeability formulae. To model fissures as equivalent parallel plate conductors, the flow between parallel plates must be defined (Snow 1965 and Wilson and Witherspoon 1970). Laminar flow of an incompressible viscous fluid between smooth, parallel plates can be expressed as

\[
v = \frac{\gamma_w d^2}{12 \mu_w} i
\]

where

\[
\begin{align*}
\gamma_w &= \text{unit weight of water} \\
\mu_w &= \text{dynamic viscosity of water} \\
d &= \text{aperture between smooth parallel plates}
\end{align*}
\]

The volume flow rate per unit width, \( q \), becomes

\[
q = \frac{\gamma_w d^3}{12 \mu_w} i
\]

Comparison of the flow velocity equation with Darcy's law indicates that the parallel plate permeability, \( k_p \), can be expressed by

\[
v = k_p i
\]

where

\[
k_p = \frac{\gamma_w d^2}{12 \mu_w}.
\]

(2) Individual Fissure Tests. The majority of the laboratory tests which have been conducted, to date, have been on individual fissures. Tests on individual fissures are perhaps the most flexible of the laboratory tests for rock mass permeability. The tests are generally conducted at various flow rates within the individual fissure and at various normal loads to simulate the in situ effective stress flow conditions. The data are used to develop
correlations for predicting the permeability under the anticipated in situ stress and hydraulic gradient conditions. The permeability measured from individual fissures provides direct input into discontinuum analyses and may be applied to continuum analyses under certain conditions. Before fissure permeability, \( k_j \), can be determined, however, the fissure roughness must be incorporated into the equation for the parallel plate permeability, \( k_p \). To accommodate the roughness, the aperture between smooth, parallel plates, \( d \), is replaced with an equivalent parallel plate aperture, \( e \). Thus, the fissure permeability is expressed by

\[
v = k_j i
\]

(3-8)

where \( k_j = \frac{\gamma_w e^2}{12 \mu_w} \). Thus, it follows that the flow rate per unit width becomes

\[
q = \frac{\gamma_w}{12 \mu_w} e^3 i
\]

(3-9)

The value of \( e \) can be determined from flow experiments by rearranging the above equation to the form

\[
e = \left( \frac{12 \mu_w q}{\gamma_w i} \right)^{1/3}
\]

(3-10)

Values for \( e \) are determined from laboratory tests on individual fissures, and equations for the laminar equivalent permeability have been developed yielding

\[
k_e = \frac{\gamma_w}{12 \mu_w} \left( \frac{e_{\text{avg}}}{b_{\text{avg}}} \right)^3
\]

(3-11)

where

- \( e_{\text{avg}} \) = average of individual values of \( e \) for fissures in the set under consideration
- \( b_{\text{avg}} \) = average of the individual spacing between fissures
Tests of individual fissures can also be analyzed for turbulent flow according to the Missbach law

\[ v^m = k'_j i \]  

(3-12)

where the volume flow rate per unit width can be expressed as

\[ q^m = k'_j e^m_i \]  

(3-13)

The turbulent coefficient of permeability thus given by

\[ k'_j = \frac{q^m}{e^m_i} \]  

(3-14)

The degree of nonlinearity, \( m \), is determined as the arithmetic slope of \( \log i \) versus \( \log q \) (Ziegler 1976). For turbulent flow analysis the equivalent parallel plate aperture, \( e \), is estimated from analysis of the linear portion of the \( q \) versus \( i \) curve given by

\[ e = \left( \frac{12\mu_w q}{\gamma_w i} \right)^{1/3} \]  

(3-15)

(3) Representative Sample Tests. Another laboratory approach to measuring permeability is to test a representative sample from a rock mass. The obvious difficulty with such tests, however, is the problem of obtaining a representative specimen of reasonable dimensions. The tests may be conducted as standard laboratory permeability tests, but on a larger scale. In addition to the standard permeability tests, small-scale pressure injection tests may be conducted on such specimens, but only a limited amount of success should be expected.

(4) Evaluation of Methods of Analysis. Model tests, as well as the testing of representative samples, have only limited application and the results are frequently subject to much speculation. Measured fissure permeability and computed equivalent permeability can be directly applied to discontinuum and continuum analyses, respectively. In general, laboratory tests are normally representative of only a very small portion of the rock mass under consideration. In addition, the results of such tests can be altered significantly by either the control or the interpretation of the parameters involved in the test. Laboratory tests, therefore, should be used as a supplement to, rather than in lieu of, field tests.
b. Interpretation from Geologic Properties. Theoretical and empirical formulae have been developed which relate permeability to geologic properties. The parameters generally required for interpretation, or computation, are the average fissure aperture, the average surface roughness of the fissures, and the average fissure spacing or number of fissures per given length. The governing assumption for any interpretation by this method is that the rock mass permeability is controlled by fissures and that the fissures may be modeled as equivalent parallel plate conductors. Extensive borehole logging or observation and mapping of exposed surfaces is required for determining the parameters to be used in the analysis.

(1) Analytical Procedures. If no laboratory tests are performed to determine an equivalent parallel plate aperture, an equivalent permeability can be estimated solely from field data. The requirements are core samples for determining surface roughness of fissures and borehole logging to determine the fissure apertures and spacing. To evaluate the surface roughness of fissures, Louis (1969) defined a surface roughness index, $S$, as

$$S = \frac{y}{a}$$

(3-16)

where

- $y$ = mean height of the asperities on the fissure walls
- $a$ = mean fissure aperture

For $S < 0.033$ equations for laminar equivalent permeability have been developed which yield

$$k_e = \frac{y_w}{12\mu_w} \left( \frac{a_{\text{avg}}}{b_{\text{avg}}} \right)^3$$

(3-17)

where $a_{\text{avg}} =$ average of the individual values of $a$. For $S > 0.033$ equations for the laminar equivalent permeability have been developed which yield

$$k_e = \frac{y_w}{12\mu_w} \frac{1}{(1 + 8.8 S_{\text{avg}}^{1.5})} \left( \frac{a_{\text{avg}}}{b_{\text{avg}}} \right)^3$$

(3-18)

where $S_{\text{avg}} =$ average of the individual values of $S$. The above equations generated by Louis (1969) are empirical and are the result of numerous pipe flow experiments and separate tests of fissures with different roughness,
modeled as openings between parallel slabs of concrete. In addition to laminar flow formulae, equations may be developed for turbulent flow for both the hydraulically smooth and the completely rough flow regimes.

(2) Evaluation of Method of Analysis. Comparisons of permeability interpreted from geologic properties with permeability measured by other methods have indicated the potential for large differences between the computed and the actual permeability. Interpretations of permeability from fissure properties are made difficult by obvious problems in measuring fissure apertures and roughness. Natural fissures can have complex surficial geometries and can only be observed in boreholes or exposed surfaces, which may be disturbed during excavation. Potentially large discrepancies are possible, due to the possibility of many fissures near the borehole being either exaggerated, constricted, or discontinuous, due to the disturbance during drilling. While interpretation of permeability from geologic properties is theoretically possible, the results should be used with caution.

c. Field Measurements. By far the most accurate and most reliable technique for determining the permeability of a rock mass is that of field testing. The use of field tests results in larger volumes of the rock mass being tested and the tests are performed under in situ conditions. Field tests have generally been limited to ground-water velocity measurements, pumping tests, and injection tests.

(1) Ground-water Velocity Measurements. The equivalent permeability can be computed for a rock mass by measuring the ground-water velocity and the hydraulic gradient when certain criteria are met. It must be assumed that steady-state horizontal flow intersects the well and flow is governed by Darcy's law. There are several techniques available for measuring ground-water velocity downhole as discussed below.

(a) Temperature Probes. The velocity of ground water moving through a borehole may be determined with the use of temperature probes or sensors. Such devices consist of a small heater strip or coil mounted beneath a thermistor. The amount of heat dissipated is a function of the ground-water velocity, and properly calibrated, the devices can sense very low velocities (below 1 ft/min). The directional components of flow may be obtained by rotation of the device or by the construction of orthogonal flow channels within the device itself.

(b) Flowmeters. When conditions of high ground-water flow rates are encountered, small horizontally mounted commercial flowmeters may be placed downhole for measuring the velocity. The direction of flow may be determined by varying the orientation of the flowmeter.

(c) Tracer Tests. Tracer tests involve the injection of an inert solution, or tracer, into an existing flow field via a borehole or well. Tracer tests are often desirable because they are passive-type tests and do not place unnatural stress conditions on the flow system. The dilution rate of the tracer at the injection well or its time of travel to another well can be used to calculate the ground-water velocity and ultimately the permeability. Detection of the tracer, or concentration measurements, can be made by either
manual or probe sampling. Generally, the probe method of sampling is desirable to avoid any disturbance to the flow system due to sample extraction. The "travel time" tracer tests normally involve large portions of the rock mass, and thus have the advantage of averaging the effects of exceptionally high- or low-permeability zones within the mass. The dilution method of testing is particularly applicable to determining permeability profiles within a single borehole by injection of the tracer into borehole sections isolated by packers. Commonly used tracers are radioisotopes, salt solution, and fluorescent dyes.

(d) Methods of Analysis. Downhole ground-water velocity measurement devices such as a temperature probe or flowmeters measure the Darcy velocity or discharge according to his original equation, expressed as

\[ v = k \frac{d}{e} \]  \hspace{1cm} (3-19)

or

\[ v = k \frac{i}{j} \]  \hspace{1cm} (3-20)

With the hydraulic gradient determined from observation wells in the area, the permeability can be computed directly. For tracer tests the seepage velocity is determined according to the equation

\[ v_s = \frac{d_w}{t_r} \]  \hspace{1cm} (3-21)

where

\[ d_w = \text{distance between injection well and observation well} \]
\[ t_r = \text{tracer travel time between wells} \]

Using the relationship

\[ v = v_s n \]  \hspace{1cm} (3-22)

\[ v_s n = k \frac{i}{j} \]  \hspace{1cm} (3-23)

\[ k = \frac{v_s n}{i} = \frac{d_w n}{t_r i} \]  \hspace{1cm} (3-24)
In analyzing a tracer dilution test, the flow velocity, \( v \), is related to the rate at which the tracer concentration diminishes within the test section of the injection well. For an assumed homogeneous isotropic porous medium, the velocity is determined from the following equation given by Lewis, Kriz, and Burgy (1966).

\[
v = \frac{\pi w_d \ln \left( \frac{C_r}{C_i} \right)}{8 t_d}
\]

where

- \( w_d \) = well diameter
- \( C_r \) = ratio of the final to the initial tracer concentration
- \( t_d \) = dilution time period

Analysis of dilution tests in fractured and fissured rock masses is made by applying the parallel plate analogy.

(2) Pumping Tests. Pumping tests have become an established means of determining the permeability of hydraulic characteristics of water bearing materials. In a pumping test, water is pumped from a well normally at a constant rate over a certain time period, and the drawdown of the water table or piezometric head is measured in the well and in piezometers or observation wells in the vicinity. Since pumping tests, as with tracer tests, involve large volumes of the rock mass, they have the advantage of averaging the effects of the inherent discontinuities, such as joints, fissures, fractures, etc. Most classical solutions for pump test data are based on the assumptions that the aquifers are homogeneous and isotropic, and that the flow is governed by Darcy's law. Applications of such solutions to interpretation of pumping tests in rock masses have resulted in varying degrees of success. For cases where the normal solutions have proven to be unsuccessful or inadequate, mathematical models have been developed which are capable of modeling the flow regime in various types of rock masses. With pumping tests, the major disadvantage is the period of time required to perform a test. Test durations of one week or longer are not unusual when attempting to approach steady-state flow conditions. Additionally, large diameter boreholes or wells are required since the majority of the conditions encountered require the use of a downhole pump. The analysis of pumping test results obtained from rock masses is generally completely analogous to the analyses used in classical soil mechanics. Since such analyses are well-documented, they will not be presented here.

(3) Injection Tests. Injection tests, which are the reciprocal of pumping tests, commonly involve the steady-state transmission of a fluid from a borehole into the surrounding medium. The permeability of a rock mass can be related to the relationship between the injection pressure and the flow rate. Equations and techniques have been developed for both the steady-state and the
unsteady-state conditions and for using either air or water for injection. Methods of analysis of pressure injection tests are presented in Appendix C.

(a) Water Pressure Tests. Water pressure tests, also known as packer tests (in Europe they are called Lugeon tests) are normally conducted by pumping water at a constant pressure into a test section of a borehole and measuring the flow rate. Borehole test sections are commonly sealed off by one to four packers, with the use of one or two packers being the most widely used technique. In comparison with a pumping test, a water pressure test affects a relatively small volume of the surrounding medium, because frictional losses in the immediate vicinity of the test section are normally extremely large. The test, however, is rapid and simple to conduct, and by performing tests within intervals along the entire length of a borehole, a permeability profile can be obtained. Additionally, the water pressure test is normally conducted in NX boreholes, and has the advantage of being conducted above or below the ground-water table.

(b) Air Pressure Tests. Air pressure tests are similar to water pressure tests except that air rather than water is used for the testing fluid. The air pressure test was developed for testing above the ground-water table and has predominantly been used for testing areas of high permeability such as those characteristic of rubblelike, fallback material adjacent to explosively excavated craters in rock. In such areas, water pressure tests have been inadequate due to an inability to provide water at a flow rate high enough to pressurize the surrounding media. Air pressure tests have an unlimited supply of testing fluid, as well as the advantage of a wide variety of high capacity air compressors. The disadvantage of such tests is that permeability equations must be modified for application to a compressible fluid and a conversion from the air permeability to a water permeability must be made to obtain usable results.

(c) Pressure Holding Tests. Pressure holding or pressure drop tests are usually conducted in conjunction with water pressure tests. The test is analogous to the falling head test used in soil mechanics; however, in rock the test section is normally pressurized to a value above that of the static head of water between the test zone and the ground surface. The pressures are normally measured in the test section with a transducer, and pressures versus time are recorded. The pressure holding test offers the advantage of being quick and simple to perform, as well as requiring significantly less water than that used in conventional constant pressure tests.

3-4. Applications of Rock Mass Permeability.

(a) Assessment of Ground-water Movements. The permeability of a dam's foundation or abutments is one of the controlling factors in the movement of ground water; therefore, a valid assessment is imperative. The rock mass permeability of the dam foundation or abutment has numerous applications, each having potentially significant impact on the design or safety of the structure.

(1) Seepage Patterns. Seepage patterns which are expected to develop after impoundment of a reservoir should be evaluated during the design phase of
a dam. Seepage patterns which do actually develop after a reservoir is in operation provide valuable data for an evaluation of the as-built performance of any cutoff or relief measures, sources and exits of seepage, and input for planning required remedial measures, such as additional cutoffs, drains, etc. In rock masses, generally, the overall trend of the seepage patterns can be determined by the continuum approach; however, in some cases, a detailed analysis of a specific seepage problem requires the use of a discontinuum approach.

(2) Flow Rates. Given the permeability and gradient, the flow rate across a given area can be computed. With the effective porosity, the rate of advance of a seepage front can be determined. Ground-water flow rates are required for determining construction dewatering requirements for both surface and subsurface excavations, as well as the seepage losses through dam foundations and abutments.

b. Foundation and Abutment Drainage Requirements. The permeability of a dam's foundation and abutments is the major factor involved in evaluating drainage requirements. Accurate and reliable rock mass permeability measurements are required in the design phase for determining the necessity for, and extent of, cutoff, vertical drains, drainage blankets, relief wells, etc. Postconstruction problems with leakage, excessive uplift pressures, etc., also require an in-depth evaluation of the flow of the water in a foundation or abutment. Evaluations of drainage or relief requirements are generally directed toward specific areas and, in most cases, while a continuum approach can give satisfactory approximations, the discontinuum analysis is required for describing the flow characteristics in details.

c. Grouting Requirements and Effectiveness. Consistent with determining the drainage requirements for a structure, the permeability is also useful for estimating the grouting requirements. In the design phase of a dam the requirements for cutoff or grouting, and drainage or relief, compliment each other and are balanced to obtain a desired seepage pattern and uplift pressure distribution beneath or within the structure. In foundations, and particularly in abutments, permeability measurements can indicate possible solution channels, faults, fissures, or other highly permeable zones which require grouting. If permeability tests are conducted at a given location, both before and after a grouting operation, an evaluation of the grouting effectiveness can be made. Such tests should be conducted as a matter of routine since they will either establish a confidence level or indicate the need for additional grouting.

d. General Considerations. The application of rock mass permeability to seepage control methods and evaluations has become increasingly useful as more complex methods of analysis evolve.

(1) Index Tests. The determination of rock mass permeability for dam foundations and abutments has historically been used more as an index test than as an absolute test. As the state of the art advances this trend is gradually changing. While index tests are a valuable tool to the experienced foundation engineer, more complex methods of analysis are becoming available.
which allow a complete evaluation of the mechanisms involved as water flows through a fractured or fissured rock mass.

(2) Continuum Analyses. The accuracy of continuum approximations of permeability is dependent upon the geology and the size of the area under consideration. In general, continuum analyses provide average permeabilities or regional permeabilities in which the assumptions of isotropy and homogeneity can be used in a gross sense. From a practical point of view the continuum approach has been and continues to be a useful tool for evaluating rock mass permeability.

(3) Discontinuum Analyses. Recently the discontinuum approach to determining permeability has become more and more promising. As techniques for determining individual fissure permeabilities advance, the availability of better modeling techniques increases, and understanding of the influences of orientation, spacing, apertures, and surficial geometries of fissures and fissure sets increase, the discontinuum approach to determining rock mass permeability becomes more reliable.
4-1. General Considerations. Seepage as used in this manual is defined as the flow of water through homogeneous saturated soil under steady-state conditions. Additionally, the soil particles, soil structure, and water are assumed incompressible and flow obeys Darcy's law. Thus transient conditions such as a wetting front or other movement of water in unsaturated soil, consolidation, and subsidence are not considered for analysis. Principles which characterize movement of energy through conducting media also apply to the movement of water through soils. Seepage has been modeled for study by using flow of electricity and heat. Both conditions are governed by Laplace's equation in homogeneous media. As explained in Chapter 2, water moves from a higher energy state to a lower energy state, and in seepage the difference in energy states is the amount of energy required to move the water through the soil, i.e., to overcome the soil's resistance to the flow of water. Chapter 4 will consider factors controlling seepage, equations describing seepage, methods of determining pressure distribution and pressures at particular points in the soil, and seepage quantities. For example, in figure 4-1(a): What is the uplift pressure at point 5? How much water will exit at point 8? How fast? Will the sand at point 8 be eroded? If the sheet pile at point 6 is removed, how will it affect pressure distribution beneath the dam?

4-2. Boundary Conditions.

a. Basis. The saturated soil which is considered for analysis must be defined by boundaries, permeability of the soil, and heads imposed upon the water. This section considers the types of boundaries which may define a particular porous soil mass considered for analysis. The nature and location of these boundaries are determined by a soils exploration program, assumptions based on engineering judgment and conditions imposed by the proposed design. Normally, simplifying assumptions are required in order to establish boundaries which will make analysis feasible. Generally, seepage analysis problems associated with dams will involve four possible types of boundaries (Harr 1962). Examples of the four general types of boundary conditions are shown in figure 4-1.

b. Impervious Boundaries. The interface between the saturated, pervious soil mass and adjacent materials such as a very low permeability soil or concrete is approximated as an impervious boundary. It is assumed that no flow takes place across this interface, thus flow in the pervious soil next to the impervious boundary is parallel to that boundary. In figure 4-1, lines AB and 1-8 are impervious boundaries.

c. Entrances and Exits. The lines defining the area where water enters or leaves the pervious soil mass are known as entrances or exits, respectively. Along these lines (O-1 and 8-G in figure 4-1(a) and AD and BE in figure 4-1(b)) are lines of equal potential; that is, the piezometric level is the same all along the line regardless of its orientation or shape. Flow is perpendicular to an entrance or exit. Entrances and exits are also called reservoir boundaries (Harr 1962).
d. Surface of Seepage. The saturated pervious soil mass may have a boundary exposed to the atmosphere and allow water to escape along this boundary, line GE, figure 4-1(b). Pressure along this surface is atmospheric. The surface of seepage may also be called a seepage face.

e. Line of Seepage. Known also as the free surface, this boundary is located within the pervious soil where water is at atmospheric pressure, line DG, figure 4-1(b). Because of capillary forces, the saturated zone of pervious soil extends slightly above the line of seepage, but this capillary zone rarely has significant influence on seepage analysis. Whereas the first two boundaries are normally defined by the geometric boundaries of the saturated porous soil mass, the line of seepage is not known until the flow distribution within the pervious soil is known. Again, as for an impervious boundary, the assumption is made that no flow takes place across the line of seepage, thus flow in the pervious soil next to this boundary is parallel to the boundary.

4-3. Confined and Unconfined Flow Problems. Two general cases of seepage are considered in this manual: confined and unconfined flow. Confined flow exists in a saturated pervious soil mass which does not have a line of seepage boundary. Figure 4-1(a) is an example of confined flow. Unconfined flow, figure 4-1(b), exists when the pervious soil mass has a line of seepage. Thus confined flow has all boundaries defined while for unconfined flow the surface of seepage and line of seepage must be defined in the analysis.
4-4. Laplace's Equation.

a. Seepage Analysis. In order to do a seepage analysis, a general model describing the phenomena of seepage must be available. Supplied with specific boundary conditions and soil properties, this model can be used to determine head and flow distribution and seepage quantities. The Laplace equation is the mathematical basis for several models or methods used in seepage analysis.

b. Basis of Laplace's Equation. Figure 4-2 shows a general seepage condition from which an element is taken. Development of Laplace's equation depends on six assumptions:

1. Heads $h_1$ and $h_2$ are constant and thus flow is steady state.
2. Water is incompressible.
3. Volume of voids does not change—soil is incompressible.
4. Flow is laminar—Darcy's law applies.
5. The element has a dimension, $dy$, into the plane of the figure which gives an element volume but no flow takes place perpendicular to the plane of the figure, i.e., the flow is two-dimensional.
6. The saturated pervious soil stratum is homogeneous. From figure 4-2(b) let:

$$v_x, v_z = \text{components of discharge velocity in } x \text{ and } z \text{ directions, respectively}$$

$$i_x = -\frac{\partial h}{\partial x} \text{ hydraulic gradient in the } x \text{ direction}$$

$$i_z = -\frac{\partial h}{\partial z} \text{ hydraulic gradient in the } z \text{ direction}$$

(The minus sign indicates that gradient is in a direction opposite increasing piezometric head.) Assumptions 1, 2, and 3 assure continuity of flow which means that water entering the element per unit of time, $q_e$ (where $q_e = v_x \, dz \, dy + v_z \, dx \, dy$) equals water leaving the element per unit of time, $q_L$ (where $q_L = v_x \, dz \, dy + \frac{\partial v_x}{\partial x} \, dx \, dz \, dy + v_z \, dx \, dy + \frac{\partial v_z}{\partial z} \, dz \, dx \, dy$). Setting $q_e$ equal to $q_L$ gives:
Using Darcy's law, \( v = ki \) and assuming the same permeability in the x and z directions:

\[
\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0
\]  

or

\[
\frac{\partial v_x}{\partial x} = -k \frac{\partial h}{\partial x}
\]

\[
\frac{\partial v_z}{\partial z} = -k \frac{\partial h}{\partial z}
\]

\( kh \) is called a potential or velocity potential and is normally given the symbol \( \phi \). Thus

\[
\phi = kh
\]

and

\[
\frac{\partial v_x}{\partial x} = -\frac{\partial \phi}{\partial x}, \quad \frac{\partial v_z}{\partial z} = -\frac{\partial \phi}{\partial z}
\]

Substituting into equation 4-1 gives

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0
\]  

which is a form of the Laplace equation for laminar, two-dimensional flow in homogeneous, isotropic, porous media. The development here follows Terzaghi 1943. Rigorous developments can be found in Bear 1972, Cedergren 1977, and Harr 1962.
4-5. Methods for Solution of Laplace's Equation. Solutions to steady-state, laminar flow, seepage problems must solve Laplace's equation. Several methods have been developed to solve exactly or approximately Laplace's equation for various cases of seepage, figure 4-3 (Radhakrishnan 1978). One of the most widely used methods, the flow net, can be adapted to many of the underseepage and through-seepage problems found in dams and other projects involving hydraulic structures. This method will be covered in detail in Section 4.6.

a. Models. Models which scale or simulate the flow of water in porous media can provide a good feel for what is occurring during seepage and allow a physical feel for the reaction of the flow system to changes in head, design geometry, and other assumptions. Appendix B contains examples of the various model types.

(1) As previously mentioned, processes which involve movement of energy due to differences in energy potential operate by the same principles as
Figure 4-3. Seepage analysis methods (from Radhakrishnan)
movement of confined ground water. These processes include electricity and heat flow which have been used as seepage analogies. Electrical analogies have proven particularly useful in the study of three-dimensional problems and in problems where geometric complexities do not allow adequate simplifying assumptions for analytical methods. "Wet" electrical analogies normally use a conducting aqueous solution or gel to model the volume of the confined, saturated, porous soil. Wet models are well suited to projects where an irregular structure penetrates a confined aquifer. By probing the gel or solution when a set potential or voltage is applied across it, electrical potential can be determined at various points of interest in the model aquifer (McAnear and Trahan 1972, Banks 1963, 1965). When field conditions can be characterized by a two-dimensional plan or section, conducting paper models may be used to inexpensively determine the effect of various configurations on the flow and pressures in the aquifer, figure 4-4 (Todd 1980).

(2) Sand models which may use prototype materials can provide information about flow paths and head at particular points in the aquifer. The sand or porous material may be placed underwater to provide a homogeneous condition, or layers of different sand sizes may be used to study effects of internal boundaries or layers. If the flow is unconfined and the same material is used for model and prototype, the capillary rise will not be scaled and must be compensated for in the model. Flow can be traced by dye injection and heads determined by small piezometers. Disadvantages include effects of layering when the porous material is placed, difficulty in modeling prototype permeability and boundary effects. Prickett (1975) provides examples of sand tank models and discusses applications, advantages, and disadvantages.

(3) Viscous flow models have been used to study transient flow (e.g., sudden drawdown) and effects of drains. This method depends on the flow of a viscous fluid such as oil or glycerin between two parallel plates and is normally used to study two-dimensional flow. As with sand models, dye can be used to trace flow lines. Construction is normally complicated and operation requires care since temperature and capillary forces affect the flow. Flow must be laminar, which can be difficult to achieve at the boundaries or at sharp changes in boundary geometry.

b. Analytical Methods.

(1) Harr (1962) explains the use of transformations and mapping to transfer the geometry of a seepage problem from one complex plane to another. In this manner, the geometry of a problem may be taken from a plane where the solution is unknown to a plane where the solution is known. While this method has been used to obtain solutions to general problems it is not frequently used for solutions to site-specific seepage problems since it requires the use of complex variable theory and proper choice of transformation functions.

(2) Pavlovsky (1936, 1956) developed an approximate method which allows the piecing together of flow net fragments to develop a flow net for the total seepage problem. This method, termed the Method of Fragments, allows rather complicated seepage problems to be resolved by breaking them into parts, analyzing flow patterns for each, and reassembling the parts to provide an
overall solution. Appendix B contains details of the Methods of Fragments based on Harr's (1962) explanation of Pavlovsky's work.

(3) Closed form solutions exist for simpler seepage conditions such as flow to a fully penetrating well with a radial source (Muskat 1946). Seepage problems associated with dams typically require approximate solutions because of complicated flow conditions.
c. **Numerical and Computer Methods.** Computer models are used to make acceptable approximations for the Laplace equation in complex flow conditions. The two primary methods of numerical solution are finite difference and finite element. Both can be used in one-, two-, or three-dimensional modeling. Several computer programs for these methods are available within the Corps of Engineers (Edris and Vanadit-Ellis 1982).

(1) The finite difference method solves the Laplace equations by approximating them with a set of linear algebraic equations. The flow region is divided into a discrete rectangular grid with nodal points which are assigned values of head (known head values along fixed head boundaries or points, estimated heads for nodal points that do not have initially known head values). Using Darcy's law and the assumption that the head at a given node is the average of the surrounding nodes, a set of N linear algebraic equations with N unknown values of head are developed (N equals number of nodes). Simple grids with few nodes can be solved by hand. Normally, N is large and relaxation methods involving iterations and the use of a computer must be applied. Appendix B provides details of this method.

(2) The finite element method is a second way of numerical solution. This method is also based on grid pattern (not necessarily rectangular) which divides the flow region into discrete elements and provides N equations with N unknowns. Material properties, such as permeability, are specified for each element and boundary conditions (heads and flow rates) are set. A system of equations is solved to compute heads at nodes and flows in the elements. The finite element has several advantages over the finite difference method for more complex seepage problems. These include (Radhakrishnan 1978):

(a) Complex geometry including sloping layers of material can be easily accommodated.

(b) By varying the size of elements, zones where seepage gradients or velocity are high can be accurately modeled.

(c) Pockets of material in a layer can be modeled.

4-6. **Graphical Method for Flow Net Construction.** Flow nets are one of the most useful and accepted methods for solution of Laplace's equation (Casagrande 1937). If boundary conditions and geometry of a flow region are known and can be displayed two dimensionally, a flow net can provide a strong visual sense of what is happening (pressures and flow quantities) in the flow region. Equation 4-2, paragraph 4.4, is an elliptical partial differential equation whose solution can be represented by sets of orthogonal (intersecting at right angles) curves. One set of curves represents flow paths of water through the porous media while curves at right angles to the flow paths show the location of points within the porous media that have the same piezometric head. The former are called flow lines, the latter equipotential lines. The flow net is a singular solution to a specific seepage condition, i.e., there is only one family of curves that will solve the given geometry and boundary conditions. This does not mean that a given problem will have only one flow net—we may choose from the family of curves different sets of curves to define the problem, figure 4-5. The relationship between the number of equipotential
drops, \( N_d \), and flow channels, \( N_f \), does not change. A brief study of figure 4-5 will provide a feel for where quantity of flow is greatest, velocity highest, and gradient highest, i.e., in the area of the porous soil nearest the sheet pile (flow channel 4, figure 4-5(a); flow channel 5, figure 4-5(b)). This section draws upon several publications which give

\[ \text{Figure 4-5. Flow net for a sheetpile wall in a permeable foundation} \]

(from U. S. Army Engineer District, Little Rock)

detailed explanation of flow net derivation and drawing instructions (Casagrande 1937; Cooley, Harsh, and Lewis 1972; Soil Conservation Service 1973; and Cedergren 1977). One of the best ways to develop an understanding of seepage and flow nets is to study well-drawn flow nets found in these and other references and to practice drawing them.

a. Assumptions for Flow Net Construction. In order to draw a flow net, several basic properties of the seepage problem must be known or assumed:

(1) The geometry of the porous media must be known.

(2) The boundary conditions must be determined (see paragraph 4.2).

(3) The assumptions required to develop Laplace's equation must hold (see paragraph 4.4b).

(4) The porous media must be homogeneous and isotropic (anisotropic conditions are dealt with in paragraph 4.7).
b. Guidelines for Flow Net Drawing. Once the section of porous media and boundary conditions are determined, the flow net can be drawn following general guidelines:

(1) Determine flow conditions at the boundaries:

(a) Flow will be along and parallel to impermeable boundaries lines BCD and FG, figure 4-5.

(b) Entrances and exits are equipotential lines, lines AB and DE, figure 4-5, with flow perpendicular to them.

(c) Flow will be along and parallel to a line of seepage--line AB, figure 4-6.

(d) Entrance and exit conditions for a line of seepage are shown in figure 4-7 under "Conditions for Point of Discharge."

This will provide a feel for the flow net.

(2) Equipotential and flow lines must meet at right angles and make curvilinear squares. Usually, it is best to make either the number of flow channels a whole number (if the number of flow channels is a whole number, the number of equipotential drops will likely be fractional).

(3) Generally, a crude flow net should first be completed and adjustments applied throughout the net rather than defining one portion since refinement of a small portion tends to shift the whole net.

(4) The initial emphasis should be on getting intersections of flow lines and equipotential lines at 90°, then shifting lines to form squares.

(5) If, in the finished flow net, either equipotential drops or flow channels end up as a whole number plus a fractional line of squares (equipotential drop or flow channel), this should not be a problem but must be used in any calculations based on the flow net. It is convenient to locate a partial equipotential drop in an area of uniform squares since this will make accurate estimation of the fraction easier.
(6) Use only enough flow lines and equipotential lines to bring out flow net definition. If more information is needed in particular areas, the squares may be subdivided into smaller squares for more detail of flow and pressure distribution.

(7) As shown in figure 4-6, equipotential line intersections at a line of seepage, line AB, and a surface of seepage or discharge face, line BC, are controlled by elevation since pressure is atmospheric along these lines. Along the discharge face BC, the equipotential lines and flow lines do not form squares since the discharge face is not a flow line or an equipotential line but a line at atmospheric pressure and changing elevation potential.

(8) Figures 4-7 and 4-8 provide some guidelines for entrances and exits and particular areas within the flow region.
For foundations, furthermost upstream and downstream flow lines and equipotential lines should intersect at or near the center of the pervious foundation.

The flow line and equipotential line nearest an angle should intersect on the bisector of the angle.

Same as (b) except for an upstream toe on an impervious foundation.

2:1 length ratios to establish shape of the "square" in a pervious foundation at the toe of an impervious fill.

2:1 length ratios used with angle bisectors to shape flow around an imbedded 90-degree angle.

2:1 length ratios to establish flow directions beneath a thin cutoff wall taken to the midpoint of the pervious stratum.

Subdivide to check odd-shaped "squares". Resulting smaller odd-shaped "squares" should have the general shape of the one subdivided.

Figure 4-8. Guides for flow net construction (from U. S. Department of Agriculture)
(9) It is helpful to lay out the boundaries which will contain the flow net in ink and use a soft pencil and eraser to develop the flow net to final form.

(10) Accuracy of squares may be checked by drawing diagonals for a square or subdividing the square by sketching an additional flow line and equipotential line orthogonal to it (ad infinitum). The diagonals should be smooth curves intersecting at right angles. Also, if the intersection of two flow lines and two equipotential lines is a square, a circle, tangent to each of the sides, may be inscribed within the square.

(11) For calculation of seepage quantity only a crude flow net is required. Accurate flow nets are required to determine pressure distribution.

4-7. Flow Net for Anisotropic Soil. Most naturally occurring soils and many man-placed soils have greater horizontal permeabilities than vertical. This affects the shape of a flow net since the flow net provides a solution to Laplace's equation which is based on the assumption of an isotropic porous media (paragraph 4.4b). To compensate for anisotropy, the dimensions of the porous media are changed by the square root of the ratio of the two permeabilities. If $k_h$ is the horizontal permeability and $k_v$ is the vertical permeability, then the horizontal dimensions of the porous media cross section are changed by a ratio of $\sqrt{\frac{k_v}{k_h}}$, e.g., if the base of a dam is 300 feet, then it would be changed by a factor of $\sqrt{\frac{k_v}{k_h}}$, or would be 300 feet times $\sqrt{\frac{k_v}{k_h}}$. The same ratio would be applied to all other horizontal dimensions to produce a transformed section. Next the flow net is drawn on the transformed section, as described in paragraph 4-6. Then the section, including the flow net, is returned to the original (true section) which produces a nonsquare flow net. Computations are made using the nonsquare flow net just as a square flow net is used for isotropic conditions. This procedure is illustrated in figures 4-9 and 4-10. In the same manner, dimensions in the vertical direction could be changed by the factor $\sqrt{\frac{k_h}{k_v}}$, square or normal flow net drawn on the transformed section, then returned to true section. Pore pressure distribution and hydrostatic uplift may be taken from either section while gradient and magnitude of seepage forces must be determined from the true section.

4-8. Flow Net for Composite Sections. Commonly, projects requiring seepage analysis involve different soils with different permeabilities, e.g., stratified foundation materials and zoned dams. Certain rules apply to flow lines, equipotential lines, and lines of seepage crossing internal boundaries between soils of different permeabilities. Figure 4-11 illustrates the deflection of flow lines and equipotential lines at interfaces. The essential principle is that the more permeable soil allows the same amount of water to flow with less restriction, thus drops in potential within the higher permeability soil will be farther apart (i.e., less energy loss in the higher permeability soil for the same length of flow as in the low permeability soil). It should be noted in figure 4-11 that when flow goes from lower permeability soil to higher permeability soil, the distance between flow lines decreases (flow channel gets smaller) and the distance between equipotential drops increases, Figures 4-12 through 4-14 are examples of flow net construction for seepage through soils of
a. True section for \( k_h = k_v \) and transformed section for anisotropic conditions.

\[ k_h = 4k_v \]

b. True section for flow net in a for \( k_h = 4k_v \).

c. True section for flow net in a for \( k_h = 9k_v \).

Figure 4-9. Flow nets constructed on transformed section and redrawn on true section (courtesy of John Wiley and Sons)
Figure 4-10. Example of technique for developing a flow net for an anisotropic foundation (from U.S. Department of Agriculture)
Figure 4-11. Deflections of flow lines at interfaces of soils having different permeabilities (courtesy of New England Waterworks Association).
a. Identify boundaries

$k_2 = 5k_1$

b. Determine deflection of flow net at interface between soils with different permeabilities.

c. Trial flow net

d. Final flow net

Figure 4-12. Flow net construction for a composite section (courtesy of John Wiley and Sons)
a. Construct flow net assuming an impermeable foundation.

b. External equipotential lines into foundation without adjusting lines of net in dam.

c. Adjust flow net until balanced.

Figure 4-13. Flow net construction for embankment on a foundation of lower permeability (courtesy of John Wiley and Sons)
a. Construct flow net assuming impermeable embankment.

b. Extend equipotentials up into dam locating initial position of line of seepage.

c. Adjust flow net to meet basic flow net requirements.

Figure 4-14. Flow net construction for an embankment on a foundation of higher permeability (courtesy of John Wiley and Sons)
differing permeabilities. In all cases, flow lines and equipotential lines maintain continuity across the interface between the soils though direction will change abruptly. Additionally, the number of flow channels must remain constant throughout the flow net. For the two examples of embankments with foundations of differing permeabilities, figures 4-13 and 4-14, the flow is more or less parallel to the interface, and the more permeable zone will dominate flow location and quantity. Because of this, the flow net can be started by assuming all flow goes through the most permeable zone. Once this flow net is drawn, it is extended into the lower permeability zone and refined to meet the general flow net criteria of paragraph 4.6. Transferring a line of seepage across the interface of soils of differing permeability, such as in a zoned dam, is more involved than transferring of flow lines and equipotential lines and will be described in Chapter 6.

4-9. Determination of Seepage Quantities, Escape Gradients, Seepage Forces, and Uplift Pressures. A flow net is a picture of seepage conditions under given geometry and boundary conditions. It explains how pressures are distributed and where flow is being directed. Coupled with the knowledge of head imposed on and the permeability of the porous media, the flow net can supply important information about stability and flow quantity in two-dimensional idealization of the real situation.

a. Seepage Quantities. Each of the complete flow channels passes an equal volume of water per unit of time, while partial channels carry a proportional flow. Each of the complete potential drops between equipotential lines is an equal portion of the total head, $h$, applied across the flow net with partial drops having a proportionally smaller part. The number of flow channels, including any partial channel, is given the symbol $N_f$ while the number of equipotential drops, including any partial drops, is given the symbol $N_d$. The ratio of $N_f/N_d$ is called the shape factor, $\beta$, which is a characteristic of the given geometry and boundary conditions and permeability ratios ($k_1/k_2$, $k_v/k_h$). Quantity of flow per unit length through the porous media can be determined by using Darcy's law, $q = kiA$ and the shape factor. Total flow is the sum of the flows through each flow channel, i.e., $q = \Sigma \Delta q = N_f \Delta q$ where $\Delta q$ is the flow through each complete flow channel. In figure 4-10, $q = N_f \Delta q = 6.65 \Delta q$. Since $\Delta h$ is the head loss between each equipotential line ($h = N_d \Delta h$) and $\Delta \xi$ is the dimension of a flow net square:

$$i = \frac{\Delta h}{\Delta \xi}$$

and from the Darcy equation:
\[ \Delta q = k \frac{\Delta h}{\Delta \ell} a \]

where \( a \) is the area of the rectangle perpendicular to the flow direction. If one side of the rectangle is one unit of length perpendicular of the plane of the flow net, and the other dimension is \( \Delta \ell \), thus \( a = \Delta \ell (1) \). This leads to:

\[ \Delta q = k \frac{\Delta h}{\Delta \ell} \Delta \ell (1) = k \Delta h (1) \]

\[ = k \frac{h}{N_d} (1) \]

Then:

\[ q = \Delta q N_f \]

\[ q = k \frac{h}{N_d} N_f (1) \]

\[ q = k h (1) \text{ or } k \]h

which gives the quantity of seepage flow for each unit of thickness of porous media perpendicular to the plane of the flow net. Figure 4-10(b) gives an example of this calculation for anisotropic seepage conditions in a dam foundation. The permeability, \( k' \), used for anisotropic conditions, \( k' = \sqrt{k_v k_h} \), is derived by Casagrande (1937).

b. Escape and Critical Gradients. The escape or exit gradient, \( i_e \), is the rate of dissipation of head per unit of length in the area where seepage is exiting the porous media. For confined flow, the area of concern is usually along the uppermost flow line near the flow exit, e.g., at the downstream edge of a concrete or other impermeable structure, figure 4-15. Escape gradients for flow through embankments may also be studied by choosing squares from the area of interest in the flow net (usually at or near the exit face and downstream toe) and calculating gradients. If the gradient is too great where seepage is exiting, soil particles may be removed from this area. This phenomenon, called flotation, can cause piping (the removal of soil particles by moving water) which can lead to undermining and loss of the structure. The gradient at which flotation of particles begins is termed the critical gradient, \( i_{cr} \). Critical gradient is determined by the in-place
Figure 4-15. Base uplift and escape gradient for concrete drop spillway (from U. S. Department of Agriculture 123)
unit weight of the soil and is the gradient at which upward drag forces on the soil particles equal the submerged weight of the soil particles, figure 4-16. The critical gradient is dependent on the specific gravity and density of the soil particles and can be defined in terms of specific gravity of solids, $G_s$, void ratio, $e$, and porosity, $n$:

$$i_{cr} = \frac{\gamma'_{w}}{\gamma_w} \frac{G_s (1 - n) \gamma_{s} - \gamma_w}{\gamma_w}$$

$$= G_s (1 - n) + n - 1 = G_s (1 - n) - (1 - n)$$

$$i_{cr} = (G_s - 1)(1 - n)$$

or, since $e = \frac{n}{1-n}$ and $n = \frac{e}{1 + e}$

$$i_{cr} = (G_s - 1) \left( \frac{n}{e} \right) = (G_s - 1) \frac{e}{1 + e}$$

$$i_{cr} = \frac{G_s - 1}{1 + e}$$

If typical values of $G_s$, $e$, and $n$ for sand are used in the above equations, $i_{cr}$ will be approximately 1. Investigators have recommended ranges for factor of safety for escape gradient, $FS_g = \frac{i_{cr}}{i_e}$ from 1.5 and 15, depending on knowledge of soil and possible seepage conditions. Generally, factors of safety in the range of 4-5 (Harr 1962, 1977) or 2.5-3 (Cedergren 1977) have been proposed.

c. Heave. In some cases, movement of soil at the downstream seepage exit may not occur as flotation followed by particle-by-particle movement. A mass of soil may be lifted initially, followed by piping. This phenomenon is called heave and occurs when the upward seepage force due to differential head equals the overlying buoyant weight of soil. Heave occurs under conditions of critical hydraulic gradient. For field conditions, the point at which minimum differential head offsets the overlying buoyant weight must be determined by judgment and calculations. Terzaghi and Peck (1967) have evaluated the factor of safety with respect to heave for a row of sheet piles. Resistance to heave
Figure 4-16. Definition of critical gradient (prepared by WES)

may be developed by placing very pervious material on the exit face, which will allow free passage of water but add weight to the exit face and thus add downward force. This very pervious material must meet filter criteria to prevent loss of the underlying soil through the weighting material.

d. Seepage Forces. Forces imposed on soil particles by the drag of water flowing between them must be considered when analyzing the stability of slopes, embankments, and structures subject to pressures from earth masses. These forces are called seepage forces. The magnitude of this force on a mass of soil is determined by the difference in piezometric head on each side of the soil mass, the weight of water, and the area perpendicular to flow. The
seepage force acts in the same direction as flow, i.e. along flow lines. Consider the seepage force on plane A-A in figure 4-16. Since flow is vertically upward, the direction of seepage force is up, the difference in piezometric head is \( h \), and the area perpendicular to flow is \( A \). The seepage force, \( F_s \), is the part of the upward forces due to differential head, \( h \), or:

\[
F_s = h \gamma_A
\]

In terms of gradient and unit volume;

\[
F_s = \frac{h}{L} \gamma_w AL = i \gamma_w V
\]

and using \( f_s \) as seepage force per unit volume:

\[
f_s = \frac{F_s}{V} = i \gamma_w
\]

Two methods of applying this force to use in stability analysis are described by Cedergren (1977) and termed the gradient method and boundary pressure method. EM 1110-2-1902 gives examples of embankment stability analyses considering seepage forces. Additionally, the effect of buoyant forces on soil mass stability must also be considered. The upward or buoyant force, \( F_b \), causing reduction in effective stress on plane A-A, figure 4-15, is the remainder of the upward forces on plane A-A:

\[
f_b = L \gamma_A
\]

e. Uplift Pressures. When seepage occurs beneath concrete or other impermeable structures or strata, the underside of this impermeable barrier is subject to a force which tends to lift the structure upward. The determination of this pressure or force is important in analyzing the stability of the structure. An example of the analysis is given in figure 4-15. Summing of the uplift pressures over the bottom area of the spillway will give the total uplift force on the structure for a stability analysis. Harr's text (1962) provides methods other than flow net construction to determine uplift pressures.
CHAPTER 5
CONFINED FLOW PROBLEMS

5-1. General Considerations. As explained in Chapter 4, confined flow exists when the saturated pervious soil mass does not have a line of seepage boundary. Impervious weirs or gravity dams on pervious soil or rock are typical projects which have confined flow conditions. This chapter will consider two of these cases. While examples of this chapter use flow nets, other methods for determining uplift and gradient such as the Method of Fragments may be used.

5-2. Gravity Dam on Pervious Foundation of Finite Depth. Figure 5-1, a copy of figure 4-15, provides an example of an impervious structure on a pervious foundation of finite depth with calculation of uplift and escape gradient. Figure 4-12 illustrates a gravity dam on a composite pervious foundation of finite depth and figure 4-10 shows an example of an impervious dam (though not a gravity dam) on a finite anisotropic foundation. A cross section of a classical gravity dam/weir, figure 5-2 indicates the effect of a partially penetrating cutoff placed beneath the upstream portion of the dam. Comparing figure 4-10(b) with figures 5-1 and 5-2 will show the reduction in gradient at the downstream toe caused by embedment of the structure in the pervious foundation. Embedment provides a longer upper flow line for a given structure width and reduces the gradient at the downstream toe of the structure. Other measures to reduce uplift and/or high gradients at the downstream toe include cutoffs beneath the downstream portion of the dam and placement of drains beneath the downstream portion of the dam.

5-3. Gravity Dam on Infinitely Deep Pervious Foundations. This case, illustrated by figure 5-3, is symmetrical (if there are no asymmetrical cutoffs or drains beneath the dam) and at large distances becomes a series of half circle arcs for flow lines and radial lines for equipotential lines. Since the upstream (entrance) and downstream (exit) boundaries extend to infinity and the foundation depth is infinite, the flow quantity is infinite. Obviously the analyst must decide the limits of the problem and calculate flow quantity based on those limits. The same manipulations and effects of embedment, cutoff, and drain described for the previous case will apply to this case.
Figure 5-1. Base uplift and escape gradient for concrete drop spillway (from U. S. Department of Agriculture 123)

<table>
<thead>
<tr>
<th>Point</th>
<th>No. of Drops</th>
<th>Head Loss</th>
<th>Uplift Pressure</th>
<th>No. of Drops (18.6 in. loss)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.5</td>
<td>6.8</td>
<td>7.2</td>
<td>5.1</td>
</tr>
<tr>
<td>B</td>
<td>6.6</td>
<td>7.6</td>
<td>8.2</td>
<td>6.5</td>
</tr>
<tr>
<td>C</td>
<td>7.8</td>
<td>8.2</td>
<td>8.8</td>
<td>7.2</td>
</tr>
<tr>
<td>D</td>
<td>8.4</td>
<td>9.0</td>
<td>9.2</td>
<td>8.0</td>
</tr>
<tr>
<td>E</td>
<td>9.1</td>
<td>9.6</td>
<td>9.8</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Escape gradient at terminate and transverse sill:

\[ V = \frac{dL}{dH} = \frac{0.93}{0.22} \]

\[ \delta = 0.95 \text{ in.} \]

\[ h = 12.8 \text{ ft} \]

\[ N_a = \frac{13.8}{13.8} = 0.93 \]

\[ N_a = 13.8; \Delta h = \frac{h}{N_a} = \frac{12.8}{13.8} = 0.93 \]

Scale (ft)
Figure 5-2. Gravity dam on pervious foundation of finite depth (courtesy of McGraw-Hill Book Company)

Figure 5-3. Gravity dam on an infinitely deep pervious foundation (prepared by WES)
CHAPTER 6
UNCONFINED FLOW PROBLEMS

6-1. Introduction. This chapter will consider unconfined flow problems for cases involving earth dams. Because of their ability to give a strong visual sense of flow and pressure distribution, flow nets will be used to define seepage. Other methods, such as transformations (Harr 1962), electrical analogy models, and numerical methods, can provide pressures and flows and be used to develop flow nets. Unconfined flow problems require the solution of flow and pressure distribution within the porous media and definition of the line of seepage boundary (phreatic surface within the dam).

6-2. Homogeneous Earth Dam on Impervious Foundation. The simplest earth dam configuration consists of a homogeneous, pervious embankment on an impervious foundation. Though rarely encountered in engineered embankments, this case will introduce general methods of defining flow in embankments.

a. Definition of Unknown Seepage Boundaries and Calculation of Flow per Unit Length of Embankment, $q$. It is desired to define the flow and pressure distribution within the embankment and total flow through the embankment. The first step is determination of the upper flow line (which is the line of seepage boundary) and the length of the seepage exit face on the downstream slope of the earth dam. This provides all necessary boundary conditions for flow net construction and complete seepage definition. The two unknown boundaries, BC and CD, figure 6-1, are a combination of an entrance condition, figure 4-7(c), BB; part of a parabola, $B_1B_2$; a smooth transition between points of tangency, $B_2C$, and a straight line discharge face along the downstream slope, CD. A parabola, shown by the dashed line, is the basic geometric member used to define the location and extent of the two boundaries. Casagrande (1937) provided the standard reference for flow through embankments while others (Harr 1962, Cedergren 1977, and others) added to and refined the basic methods. Figure 6-2 provides the nomenclature and formulas for drawing the line of seepage and exit face and determining the quantity of seepage per unit length of embankment, $q$. In a given problem, embankment geometry and head water elevation provide values for $h$, $m$, and $\alpha$ which allow location of points A and B and determination of distance, d, as shown in figure 6-2.

![Figure 6-1. Line of seepage, BC, and seepage exit face, CD, for a homogeneous earth dam on an impermeable foundation (prepared by WES)](image-url)
Figure 6-2. Determination of line of seepage and seepage exit face for embankments on impervious foundations (adapted from New England Waterworks Association)
b. After this is done one of the four methods shown in figure 6-2 and explained below can be used to determine the location of the exit face CD and the line of seepage BC.

(1) \( \alpha < 30^\circ \) Schaffernak-Van Iterson. The two formulas for this method given in figure 6-2 assume gradient equals \( dy/dx \) and allow direct determination of \( a \) and \( q \). Construction of basic parabola shown in figure 6-3 is the first step in determining the upper line of seepage (Casagrande 1937). From embankment geometry and headwater height, point A is located. \( d \) and \( y \) are determined by scribing an arc, with radius DA through point E. Then the point of vertical tangency of the basic parabola, \( F \), is determined. Line AG, parallel to the embankment base and horizontal axis of the parabola, is drawn and divided into an equal number of segments (6 in the case in figure 6-3). Line GF, the vertical tangent to the parabola, located at \( y_0/2 \) from the downstream toe of the embankment is divided into the same number of equal segments as line AG. The points dividing line AG into segments are connected with point \( F \). The intersection of these lines with their counterpart lines drawn from the points on line GF define the parabola. Thus the basic parabola, dashed line A-F, is defined. The upstream portion of the line of seepage, dotted line BH, is drawn by starting at point B perpendicular to the upstream slope (since the upstream slope is an equipotential line and the line of seepage is a flow line) and continuing downstream to make line BH tangent to the basic parabola at point \( H \) which is selected based on judgment. This is an entrance condition as shown in figure 4-7. The central portion of the line of seepage is along the basic parabola while the downstream portion is a smooth transition from the basic parabola to tangency with the downstream slope at point \( C \). Point \( C \) is located a distance \( a \) from the downstream toe as determined by the equation for Schaffernak-Van Iterson shown in figure 6-2. With all seepage boundaries known and using the rules of Chapter 4, a flow net may be constructed within the boundaries as shown in figure 6-4. This figure points out the important basic flow net requirement that all equipotential lines intercept the line of seepage and exit face at points with equal vertical separation (in this case \( H/10 \) apart).
Figure 6-4. Seepage through a permeable embankment underlain by an impermeable foundation (prepared by WES)

(2) $\alpha < 90^\circ$ L. Casagrande. The gradient assumption for this method is $i = \frac{dy}{ds}$ where $s$ is the distance along the line of seepage, and allows greater accuracy than Schaffernek-Van Iterson method for steeper downstream slopes. Use of the equations in figure 6-2 and the same general procedures used for the Schaffernek-Van Iterson method apply for $\alpha^t s$ up to $60^\circ$. For $60^\circ < \alpha < 90^\circ$, since $a$ and $s$ are interdependent, the location of point C (or distance $a$) must be estimated to determine the value of $s_o = \overline{AC} + \overline{CD}$, then distance $a$ calculated. This procedure is repeated until there is satisfactory agreement between the portion of the distance $s_o$ as measured and $a$ as calculated. Thus the seepage boundaries are established allowing flow net construction.

(3) $\alpha = 180^\circ$ Kozeny. For this special case Kozeny described a solution adapted by Casagrande (1937). Figure 6-5 illustrates the nomenclature and construction method for this case. Embankment geometry, $h$, and drain location control construction of the basic parabola. For this case the seepage face is the distance $a_o$ and the correction $\Delta a$ is not used. Again with boundary definition, the flow net can be drawn.

(4) $30^\circ < \alpha < 18.0^\circ$ A. Casagrande. After study of model experiments and construction of flow nets for various $\alpha^t s$, A. Casagrande (1937) developed a curve, figure 6-6, which relates $a$ to the ratio, $c = \frac{\Delta a}{a + \Delta a}$. Construction of the basic parabola is the first step in this procedure. The point, $C_o$, as shown in figure 6-2, where the basic parabola intercepts the downstream slope is determined and distance $a + \Delta a$ is measured. Knowing $\alpha$, $C$ can be found in figure 6-6 and $\Delta a$ calculated. Information is then sufficient to draw the line of seepage and discharge face, determine $q$, and construct the flow net. Casagrande (1937) provides a procedure for the condition of tailwater on the downstream slope:

For the comparatively rare case in which the presence of tailwater must be considered in the design, the determination of the line of seepage and of the quantity can be performed by dividing the dam horizontally at tailwater level into an upper and lower section. The line of seepage is determined for the upper section in the same
Figure 6-5. Construction of basic parabola and seepage line for \( \alpha = 180^\circ \) Kozeny (courtesy of New England Waterways Association\textsuperscript{151})

Figure 6-6. \( c \) vs \( \alpha \) (courtesy of New England Waterworks Association\textsuperscript{151})

NOTE: POINTS WERE FOUND BY GRAPHICAL DETERMINATION OF FLOW NET.
manner as if the dividing line were an impervious boundary. The seepage through the lower section is determined by means of Darcy's law, using the ratio of the difference in head over the average length of path of percolation as the hydraulic gradient. The total quantity of seepage is the sum of the quantities flowing through the upper section and the lower section. The results obtained by this rather crude approximation agree remarkably well with the values obtained from an accurate graphical solution.

Harr (1962) explains an additional method, known as Pavlovsky's solution, for determining $a_0$ and $q$ for the case of a homogeneous, pervious embankment on an impervious foundation. Pavlovsky analyzed the embankment by dividing it into three zones, writing an equation for $q$ in each of the zones and, by assuming continuity of flow, equating the three equations for $q$. Figure 6-7 provides the nomenclature for Pavlovsky's solution. The embankment is divided as shown with Zone I between the upstream slope and a vertical line at the intersection of the crest and upstream slope ($y$ axis), Zone II between the $y$ axis and a vertical line at the intersection of the line of seepage with the downstream slope, and Zone III which is composed of the remainder of the downstream toe. Pavlovsky assumed horizontal flow in each zone and wrote the basic equation $q = kIA$ for each zone using the nomenclature of figure 6-7. The equations for each zone are:

**Zone I**

$$q_1 = k_1 \frac{(h_w - h_1)}{\cot \beta} \ln \left( \frac{h_d}{h_d - h_1} \right)$$ (6-1)
Zone II

\[ q_{II} = \frac{k\left[h_1^2 - (a_0 + h_o)^2\right]}{2b + 2[h_d - (a_0 + h_o)]\cot\alpha} \]  \hspace{1cm} (6-2)

Zone III

for \( h_o > 0 \)

\[ q_{III} = \frac{ka_o}{\cot\alpha} \left[ 1 + \ln\left(\frac{a_o + h_o}{a_o}\right) \right] \]  \hspace{1cm} (6-3)

for \( h_o = 0 \)

\[ q_{III} = \frac{ka_o}{\cot\alpha} \]  \hspace{1cm} (6-4)

It is assumed that \( a\), \( \beta\), \( b\), \( h_d\), \( h_w\), \( h_o\), and \( k\) are known for a given problem, thus since \( q_1 = q_{II} = q_{III} = q \) (continuity of flow, steady state conditions) only \( a_o\), \( h_1\), and \( q \) are unknown. This analysis provides three equations, (6-1), (6-2), and (6-3), or (6-4), and three unknowns. The equations may be solved in a number of ways. One method for \( h_o = 0 \) is to equate (6-1) and (6-4) and solve for \( a_o \) then equate (6-2) and (6-4) and solve for \( a_o \) :

\[ a_o = \frac{\cot\alpha}{\cot\beta} (h_w - h_1) \ln\left(\frac{h_d}{h_d - h_1}\right) \]  \hspace{1cm} (6-5)

\[ a_o = \frac{b}{\cot\alpha} + h_d - \sqrt{\left(\frac{b}{\cot\alpha} + h_d\right)^2 - h_1^2} \]  \hspace{1cm} (6-6)

then a plot of \( a_o \) versus \( h_1 \) may be made of equations (6-5) and (6-6). The intersection of the two curves representing (6-5) and (6-6) is the value of \( a_o \) and \( h_1 \) for solution. Equation (6-4) will then provide \( q \). An example from Harr (1962) is provided in figure 6-8.
Example: Determine the quantity of seepage, $q$, $a_o$, and $h_1$, using Pavlovsky's solution for the dam shown above. Compare with Schaffernak-Van Iterson and L. Casagrande Method, figure 6-2.

A. Pavlovsky's solution

For this case: $\cot \alpha = \cot \beta = 3$ \hspace{1cm} $b = 20'$ \hspace{1cm} $h_d = 80'$ \hspace{1cm} $h_w = 70'$ \hspace{1cm} $h_o = 0$ \hspace{1cm} $k = 0.002$ ft/min.

For assumed values of $h_1$ in equations (6-5) and (6-6) the resulting values for $a_o$ are given in the following tabulation and plotted in the accompanying graph.

<table>
<thead>
<tr>
<th>$h_1$</th>
<th>$a_o$</th>
<th>$h_1$</th>
<th>$a_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>19.6</td>
<td>50</td>
<td>15.9</td>
</tr>
<tr>
<td>52.5</td>
<td>18.7</td>
<td>52.5</td>
<td>17.7</td>
</tr>
<tr>
<td>55</td>
<td>17.4</td>
<td>55</td>
<td>19.7</td>
</tr>
<tr>
<td>60</td>
<td>13.9</td>
<td>60</td>
<td>24.1</td>
</tr>
<tr>
<td>65</td>
<td>8.4</td>
<td>65</td>
<td>29.3</td>
</tr>
</tbody>
</table>

Figure 6-8. Example problem of homogeneous embankment on an impervious foundation comparing solutions (Continued) (prepared by WES)
From equation (6-4)

\[
q = \frac{0.002 \text{ ft/min} \times 18.3 \text{ ft}}{3} = 0.0122 \text{ ft}^3/\text{min per ft of embankment length}
\]

\[a = 3a_o = 54.9 \text{ ft}\]

B. Schaffernak-Van Iterson solution:

Since \(m = 3(70 \text{ ft}) = 210 \text{ ft}\)

\[d = 0.3(210 \text{ ft}) + 3(10 \text{ ft}) + 20 \text{ ft} + 3(80 \text{ ft}) = 353 \text{ ft}\]

\[h_w = 70 \text{ ft}\]

\[a = \frac{353 \text{ ft}}{\cos 18^\circ 26'} - \sqrt{\frac{(353 \text{ ft})^2}{\cos^2 18^\circ 12'} - \frac{(70 \text{ ft})^2}{\sin^2 18^\circ 26'}}\]

\[a = 73 \text{ ft}\]

\[q = (0.002 \text{ ft/min})(73 \text{ ft})(\sin 18^\circ 26')(\tan 18^\circ 26')\]

\[q = 0.015 \text{ ft}^3/\text{min per ft of embankment length}\]

C. L. Casagrande solution:

\[s_o = \sqrt{(353 \text{ ft})^2 + (70 \text{ ft})^2} = 360 \text{ ft}\]

\[a = 360 \text{ ft} - \sqrt{(360 \text{ ft})^2 - \frac{(70 \text{ ft})^2}{\sin^2 18^\circ 26'}}\]

\[a = 76 \text{ ft}\]

\[q = (0.002 \text{ ft/min})(76 \text{ ft} \sin^2 18^\circ 26')\]

\[q = 0.015 \text{ ft}^3/\text{min per ft of embankment length}\]

Figure 6-8 (Concluded)
6-3. **Earth Dam with Horizontal Drain on Impervious Foundation.** Figure 6-5 presents this case for a homogeneous embankment. However, since most earth dams are built in horizontal layers, they very likely have a stratified structure which may allow considerable flow to bypass the horizontal drain, figure 6-9. The difference in vertical and horizontal structure also causes differences in horizontal and vertical permeabilities. This can strongly affect the location of the upper line of seepage, figure 6-10, which affects stability considerations and methods of controlling seepage.

![Figure 6-9. Schematic of effect of earth dam stratification on flow of water through embankment with horizontal drain (prepared by WES)](image)

6-4. **Earth Dam with Toe Drain on Impervious Foundation.** Toe drains are another method of controlling the line of seepage, figure 6-11. Again the effects of anisotropy must be considered, figure 6-12. The geometry of the embankment and the toe drain, height of reservoir, and the degree of anisotropy will control the location of the line of seepage.

6-5. **Earth Dam with Vertical or near Vertical Horizontal Drains on Impervious Foundation.** One very effective method of intercepting horizontal flows due to stratification of the embankment, figure 6-9, is the incorporation of an inclined or vertical drain into the central portion of the embankment, figure 6-13. This seepage analysis of a zoned, anisotropic embankment assumed the rockfill to have infinite permeability with respect to the core materials and used the method recommended by A. Casagrande for drawing the parabola to determine the upper line of seepage. The interface of the core and inclined drain is used as the downstream slope for the seepage face since the drain has a much higher permeability than the core material. Provision must be made in sizing the drain to pass all the water coming out of the core without building up a tailwater on the downstream slope of the core. It can be noted that the designers of this example used \( m/3 \) instead of 0.3\( m \) to determine the interception point of the parabola and headwater elevation and the formula

\[
a + \Delta a = \frac{y}{1 - \cos a}
\]

which can be derived trigonometrically, to determine \( a + \Delta a \). The geometry of the embankment and toe drain, height of reservoir, and the degree of anisotropy will control the location of the line of seepage.

6-6. **Flow Net for a Composite Zoned Dam.** If differences in permeabilities between zones in a zoned dam are great enough (e.g. 100 to 1000 times or more) the more permeable zone may be considered to have infinite permeability relative to the less permeable zone for purposes of seepage analysis. In the seepage analysis example of figure 6-13 the upstream rockfill and the inclined
Figure 6-10. Effect of difference in horizontal and vertical permeability (anisotropy) on location of line of seepage within an embankment with a horizontal drain (after U. S. Department of Agriculture)
**TRANSFORMATION**

\[ T = \frac{\sqrt{T^2 + T^4}}{1 + T^2} = 0.6 \]

**TRUE SECTION**

**DETERMINATION OF PHREATIC LINE**

Pool of Spillway Crest EL 837', Tailwater EL 806', Maximum Net Head = 131'

Assuming the embankment rests on an impervious foundation. As in the case of the left abutment, the position of the phreatic line is determined after \( A \). Casagrande. The above assumption pertains to the determination of the phreatic line only which leads to more conservative design.

\[ Y = \frac{1}{\cos A} = \frac{1}{0.8586} = 0.5261 \]

\[ Y \tan A = 2.2 = 2 \times 67.1 \]

\[ \cos A = 0.447 \]

\[ A = 78.75^\circ \]

\[ A = 85.86^\circ \]

\[ 2 \times 78.75^\circ = 157.5^\circ \]

\[ 2 \times 85.86^\circ = 171.72^\circ \]

\[ A = 0.32 \times 85.86^\circ = 27.48^\circ \]

\[ A = 0.32 \times 85.86^\circ = 27.48^\circ \]

**EMBANKMENT SEEPAGE**

For non-isotropic soil, the effective coefficient of permeability

\[ K = K_v K_h \]

Since the embankment has an impervious fill material is non-isotropic.

\[ K = \frac{0.821 \times 10^{-6}}{0.821 \times 10^{-6}} \text{ CM SEC} \]

\[ K = 8.64 \times 10^{-9} \text{ CM SEC} \]

\[ q \text{ FOOT} = 1148 \times 10^{-9} \times 131 = 15.04 \times 10^{-7} \text{ FTF SEC} \]

When \( H = 131 \) FT.

\[ q = \frac{15.04 \times 10^{-7}}{15.04} = 1.0 \times 10^{-7} \text{ FTF SEC} \]

**Figure 6-13.** Seepage analysis for Kettle Creek earth embankment, Kettle Creek, Pa. (from U.S. Army Engineer District, Baltimore)
TRANSFORMED SECTION

FLOW NET

LEFT ABUTMENT-STATION 17+00

TRANSFORMATION

K' = \sqrt{K_v K_H}, \text{ TRANSFORMATION FACTOR} = \frac{K_v}{K_H} \left(\frac{1}{2}\right)^{\frac{1}{2}} \cdot 0.5

TRANSFORMED SECTION HORIZONTAL DIMENSION = 0.5 TRUE SECTION HORIZONTAL DIMENSION

DETERMINATION OF PHREATIC LINE

POOL OF SPILLWAY CREST EL 937.0, TAILWATER EL 806.0, MAXIMUM NET HEAD = 131 ft.

ASSUMING THE EMBANKMENT RESTS ON IMPERVIOUS FOUNDATION, AS IN THE CASE OF THE LEFT ABUTMENT, THE POSITION OF THE PHREATIC LINE IS DETERMINED AFTER A. CASAGRANDE. THE ABOVE ASSUMPTION PERTAINS TO THE DETERMINATION OF THE PHREATIC LINE ONLY WHICH LEADS TO MORE CONSERVATIVE DESIGN.

\[ h = \frac{1}{2} \left( \frac{b_1^2 + b_2^2}{b_1} \right) \]

\[ \alpha = \arctan \left( \frac{b_1}{b_2} \right) \]

\[ \alpha = 85.86, \Delta \approx 0.32 \text{ (AFTER A. CASAGRANDE)} \]

\[ \Delta e = 0.32 \times 85.86 = 27.48' \]

\[ \gamma = \frac{h}{1 + \cos \alpha} \]

\[ \gamma = 47.48 \]

\[ \delta = \frac{\gamma}{1 - \cos \alpha} \]

\[ \Delta = 0.447 \]

EMBANKMENT SEEPAGE

FOR NON-ISOTROPIC SOIL THE EFFECTIVE COEFFICIENT OF PERMEABILITY

\[ K' = K_v K_H, \text{ SINCE THE EMBANKMENT IMPERVIOUS FILL MATERIAL IS NON-ISOTROPIC, } K' = \sqrt{K_v K_H} = \sqrt{0.4 \times 10^{-6}} \cdot 1.6 \times 10^{-6} \text{ CRT/SECKM} \]

\[ K' = \frac{\sqrt{0.4 \times 10^{-6}}}{0.8 \times 10^{-6} \text{ CM/SEC OR } 2.25 \times 10^{-9} \text{ FT/SEC}} \]

\[ \text{SEEPAGE QUANTITY, q/FOOT} = \frac{N_1}{N_0} K' H \]

\[ q/FOOT = 11.48 \times 10^{-9} \times 131 = 15.04 \times 10^{-7} \text{ FT3/SEC} \]

Concluded
drain are considered infinitely permeable with respect to the central earth portion of the embankment. In some cases dams may contain adjoining zones which, have relatively small but marked differences in permeability and it may be desired to accurately analyze the flow through these zones. Equipotential lines and flow lines, with the exception of the upper line of seepage, will cross the interface of the zones in the same manner as given for confined composite sections in Chapter 4, but the location of the upper line of seepage (phreatic line) must first be determined. Once its location is determined the upper line of seepage transfers between regions of different permeabilities in the manner shown in figure 6-14. In determining the location of the upper seepage line, the essential principle to remember is that the flow rate must be the same through each zone. That is, for a unit depth of embankment perpendicular to the plane of the flow net, Q must be the same for each zone. More permeable zones require less gradient and/or cross-sectional area to pass the flow transmitted to them from less permeable zones. This idea can be seen in the example and instructions taken from Cedergren 1977.

![Figure 6-14](image_url)  

**Figure 6-14.** Transfer of line of seepage between regions of different permeability (courtesy of New England Waterworks Association)

a. Locate the reservoir level and the tailwater level, noting the difference in level as h and dividing h into a convenient number of equal parts of increments \( \Delta h \). Draw a series of light horizontal guide lines (head lines) at intervals of \( \Delta h \) across the downstream part of the section (figure 6-15a).
b. Guess a trial position for the phreatic line in both zones and draw a preliminary flow net as shown in figure 6-15a, making squares in zone 1 and rectangles in zone 2. Make the length-to-width ratios of all of the rectangles in zone 2 approximately equal by adjusting the shape of the saturation line, using an engineer's scale to measure the lengths and widths of the figures. When this step is completed, the trial flow net should be reasonably well drawn. It should satisfy the basic shape requirements of a flow net, but the length-to-width ratio of the shapes in zone 2 probably will not satisfy \( \frac{c}{d} = \frac{k_2}{k_1} \). Although the flow net has been drawn for a composite section, the ratio of \( k_2/k_1 \) probably does not equal the \( k_2/k_1 \) ratio originally assumed for the section.

![Flow net diagram](image)

a. First trial-flow net (not correct)

![Flow net diagram](image)

b. Completed flow net (correct)

Figure 6-15. Method for constructing flow nets for composite sections (courtesy of John Wiley and Sons^155)

c. Calculate the actual ratio of \( k_2/k_1 \) for the trial flow net just constructed. To make this important check proceed as follows:

1. Count the number of full flow channels between any two adjacent equipotential lines in zone 1 and call this number \( n_{f-1} \). In the trial flow net in figure 6-15a, \( n_{f-1} = 4.0 \).

2. Count the number of full flow channels between any two adjacent equipotential lines in zone 2 and call this number \( n_{f-2} \). In figure 6-15a, \( n_{f-2} \) is equal to the width-to-length ratio of the figures in zone 2, \( d/c \), and equal to 0.5.

3. The actual value of \( k_2/k_1 \) for the trial flow net in figure 6-15a can now be determined from the equation
\[
k_2 = k_1 \frac{n_{f-1}}{n_{f-2}}
\]

or

\[
\frac{k_2}{k_1} = \frac{n_{f-1}}{n_{f-2}}
\]

(4) If the calculated \(k_2/k_1\) ratio is too high, the saturation line in zone 2 is too low and must be raised. If the calculated \(k_2/k_1\) ratio is too low, the saturation line in zone 2 is too high and must be lowered. Raise or lower the general level of the saturation line in zone 2 as indicated and construct another trial flow net.

(5) Repeat steps (1) through (4) until a flow net of the desired accuracy is obtained. (Usually a few trials will be sufficient.) By applying the above equation to the first trial flow net in this example (figure 6-15a) \(k_2/k_1 = 4.0/0.5 = 8.0\). Because the ratio of \(k_2/k_1\) for this example was assumed to be 5, the \(k_2/k_1\) ratio of the trial flow net is too high; hence the general level of the saturation line in zone 2 is too low and must be raised. For the second trial flow net (figure 6-15b) \(n_{f-1} = 3.5\) and \(n_{f-2} = 0.7\).

The calculated ratio of \(k_2/k_1 = 3.5/0.7 = 5.0\), the value originally assumed. The above equation may be derived by recalling that the quantity of seepage in zones 1 and 2 (figure 6-15) must be equal. Using

\[
q = kh \frac{n_f}{n_d}
\]

in zone 1, \(q = k_1 h (n_{f-1}/n_d)\), and in zone 2, \(q = k_2 h (n_{f-2}/n_d)\). For a given head \(h\), \(q \sim k_1 (n_{f-1}/n_d) \sim k_2 (n_{f-2}/n_d)\) and \(q/n_d \sim k_1 n_{f-1} \sim k_2 n_{f-2}\). Therefore \(k_1 n_{f-1} = k_2 n_{f-2}\) and \(k_2 = k_1 (n_{f-1}/n_{f-2})\). This expression can be used for determining the permeability ratios \(k_2/k_1\), \(k_3/k_2\), and so on, for any composite flow net being examined for accuracy. It is an essential criterion to be used in constructing accurate flow nets for composite sections. Figure 6-16 illustrates the additional detail which can be obtained by further subdivision of the flow net. This may be necessary for a stability analysis or other reasons. Another example provided in figures 6-17 and 6-18 illustrates a flow net for a zoned dam on an impervious foundation; figure 6-17 is the transformed section for the true section of figure 6-18 (Cedergren 1975 and U. S. Army Engineer District, Sacramento 1977).
6-7. **Zoned Earth Dam on Pervious Foundation.**

a. Zoned embankments on pervious foundations may require rather involved seepage analyses unless simplifying assumptions are made. This is particularly true when relatively small permeability differences exist between adjacent zones and foundation and it is desired to consider these differences in permeability. Figure 6-19 provides an example of such an analysis (U. S. Army Engineer Waterways Experiment Station 1956b). Development of the flow net required use of a number of principles and methods explained in Chapter 4, and previous portions of Chapter 6, e.g., dimensional changes due to anisotropy, upper seepage lines and other flow lines crossing the interface of materials having different permeabilities and composite sections. It should be noted that several assumptions were made.

(1) The flow net for the foundation was drawn independently of the embankment, i.e. considering the embankment to be impermeable, and assuming the foundation to be isotropic.

(2) The embankment flow net reflects the influence of foundation flow net in the location of equipotential and flow lines and is drawn for anisotropic conditions.
Figure 6-17. Buchanan Dam flow net studies full reservoir-Sta 9+50 D/S transition fill - $k_h = 16 k_v$ transformed section (from Cedergren and U. S. Army Engineer District, Sacramento 1977)
Figure 6-18. Buchanan Dam flow net studies full reservoir - Sta 9+50 D/S transition fill - $k_h = 16 k_v$ true section (from Cedergren and U. S. Army Engineer District, Sacramento 1977).
Figure 6-19. Seepage analysis, Blakely Mountain Dam, Arkansas (from U. S. Army Engineer Waterways Experiment Station)
Figure 6-19. Concluded
(3) The flow nets were constructed assuming a tailwater elevation of 388 ft (approximate elevation of the base of the embankment) while seepage quantities calculations assumed a tailwater elevation of 400 ft as shown on the cross sections (i.e., $h$ for calculation of $Q$ is 192 ft).

(4) For calculation of $Q$ in figure 6-19(b) the flow nets were considered to be separate but to have the same number of equipotential drops.

b. This example provides evidence that a large permeable member of the embankment-foundation material controls the quantity of flow. In this case the relatively impermeable embankment materials needed to be considered in order to determine the upper seepage line, the extent of saturated materials, and the expected pore pressures within the embankment. Seepage for this problem might also be evaluated using a finite element computer program.
CHAPTER 7
SEEPAGE TOWARD WELLS

7-1. Use of Wells. Wells are used in a variety of ways to control seepage. They may be placed on the landward side of water retention structures to reduce pressure at the lower boundary of impervious strata. Wells are also used to maintain dry conditions in excavations during construction. In addition to seepage control, well pumping tests serve as an accurate means of field determination of permeability (see Chapter 2).

7-2. Analysis of Well Problems. The graphical flow net technique described in Chapter 4 or the approximate methods described in Appendix B can be used in the analysis of well problems. However, formulas obtained from analytical solutions to well problems are the most common methods of analysis.

a. Flow Nets. An example of a flow net for a simple flow problem is shown in figure 7-1. The flow between flow lines is given by (Taylor 1948)

\[ \Delta Q = 2\pi k \Delta h \frac{rb}{l} \]  

(7-1)

where

- \( k \) = permeability (L/T)
- \( \Delta h \) = total head loss between equipotential lines (L)
- \( r \) = distance from well (L)
- \( b \) = dimension of element in Z direction (L)
- \( l \) = dimension of element in \( r \) direction (L)

As for a plan flow net, \( \Delta Q \) and \( \Delta h \) must be the same for all elements within the net. Thus \( rb/l \) is a constant. When drawn in plan view (figure 7-1b) the flow net consists of square elements as in the plane case described in Chapter 4. When drawn in profile (figure 7-1c) the elements' aspect ratios (b/R) are proportional to the radial distance \( r \) and are therefore not squares. Thus, graphical construction of flow nets for radial flow problems is generally not practical except for cases where the water bearing has a constant thickness and only the plan view of the net is required.

b. Approximate Solutions. The numerical and analog methods described in Appendix C can be used for problems involving complicated boundary conditions. Electrical analog methods are especially advantageous as most complicated well problems cannot be idealized in two dimensions.
c. Analytical Formulas. The analysis of flow to a single well can often be solved by analytical methods. Also, the analysis of flow to multiple wells and many problems involving complicated boundary conditions can be solved by superposition of solutions for single well problems. Analytical solutions can be obtained for nonsteady flow problems.

7-3. Basic Well Equations for Steady State Flow. Steady flow conditions exist when the well flow rate and piezometric surface do not change with time. If the regional piezometric surface does not fluctuate, steady state conditions are achieved by pumping from a well at a constant rate for a long time period. Design of wells for seepage control are often based on computations assuming steady state conditions.
a. Artesian Conditions. When significant flow to a well is confined to a single saturated stratum, the problem can be idealized as shown in Figure 7-2a. An artesian condition exists when the height h of the piezometric surface lies above the top of the water bearing unit b. If the properties of the soil are constant in all directions from the well, the discharge Q from the well must be equal to the flow through a cylinder defined by the radius r, height b, and differential thickness dr. Thus from Darcy's law (equation 3-3), the flow can be written as

\[ Q = k \frac{dh}{dr} 2\pi rb \]  

(7-2)

where

- \( Q \) = constant discharge from well (L^3/T)
- \( k \) = coefficient of permeability (L/T)
- \( \frac{dh}{dr} \) = hydraulic gradient along radius (L/L)
- \( r \) = radius of cylinder (L)
- \( b \) = thickness of aquifer (L)

Upon integrating equation 7-2, the relationship between r and h is found.

\[ h = \frac{Q}{2\pi kb} \ln r + \text{constant} \]  

(7-3)

The constant can be determined by specifying that at the radius \( r_e \), the total head h is equal to a known head H, the total head that existed before starting discharge from the well. That is,

\[ h = H \text{ for } r = r_e \]  

(7-4)

Inserting equation 7-4 into equation 7-3, the constant term is found to be:

\[ \text{constant} = H - \frac{Q}{2\pi kb} \ln r_e \]
a. HORIZONTAL AQUIFER CONFINED BETWEEN IMPERVIOUS STRATA (ARTESIAN FLOW)

b. HORIZONTAL UNCONFINED AQUIFER (GRAVITY FLOW)

c. COMBINED AND CONFINED AND GRAVITY FLOW

Figure 7-2. Radial flow to horizontal aquifers (courtesy of John Wiley and Sons)
By substituting the constant term into equation 7-3 and combining logarithmic terms, the well equation for confined flow is obtained.

\[ H - h = \frac{Q}{2\pi k B} \ln \frac{r_e}{r} \]  

(7-5)

The distance \( r_e \) is often defined as the radius beyond which the well has no influence or radius of influence.

b. **Gravity Flow Conditions.** Flow to a well under gravity (figure 7-2b) differs from the confined flow problem in the important aspect that the height B of the differential cylinder is equal to the variable h. Thus, equation 7-2 must be written as:

\[ Q = k \frac{dh}{dr} 2\pi rh \]  

(7-6)

which upon integration and substitution of boundary condition gives

\[ h^2 = \frac{Q}{\pi k} \ln r + \text{constant} \]  

(7-7)

The constant term can be evaluated from the condition at the radius of influence \( r_e \) as was done in equations 7-4 and 7-5. The constant term is given by:

\[ \text{constant} = H^2 - \frac{Q}{\pi k} \ln r_e \]

which when substituted back into equation 7-7 gives the well equation for gravity flow

\[ H^2 - h^2 = \frac{Q}{\pi k} \ln \frac{r_e}{r} \]

Development of equation 7-6 is based on the Dupuit assumption (Chapter 4) which limits the applicability of equation 7-7 to those cases where the slope of the piezometric surface is small (less than 5 percent). The error is greatest in the vicinity of the well.
c. Combined Artesian and Gravity Flow. When drawdown of the potentiometric surface becomes large near the well, combined gravity and confined conditions can occur (figure 7-2c).

d. Flow to Well Groups (Method of Superposition). The piezometric surface $h$ caused by discharge from a group of wells can be determined by superimposing the solution for the individual wells given by either equation 7-3 or 7-6. For multiple wells, flow cannot be idealized by concentric cylinders and the problem must be stated in terms of the plan coordinates $x$ and $y$ (figure 7-3b). By noting that for a well located at $(x_i, y_i)$, $r_i^2 = (x - x_i)^2 + (y - y_i)^2$ a general well equation can be written as

$$\phi_i = q_i \& n \, r_i + C_i$$  \hspace{1cm} (7-8)

where

- $\phi_i = $ potential required at point $(x,y)$ to sustain a discharge $Q_i$ from a well at $(x_i, y_i)$
  - $= h_i$ (confined flow)
  - $= h_i^2$ (unconfined or gravity flow)
- $q_i = $ intensity factor
  - $= Q_i/4\pi kB$ (confined flow)
  - $= Q_i/(\pi k)$ (unconfined or gravity flow)
- $C_i = $ constant

The head distribution $\phi(x,y)$ can be determined by summing the individual $\phi_i$. As the sum of the constants $C_i$ is a constant, the multiple well equation can be written as

$$\phi(x,y) = \sum_{i=1}^{n} q_i \& n \, r_i + \text{constant}$$  \hspace{1cm} (7-9)

where $n$ is the number of wells. The constant is determined from a known value of $\phi$ at a specified location. For example, the superposition formula
a. FLOW BETWEEN TWO WELLS

b. COORDINATE SYSTEM FOR FLOW TO MULTIPLE WELLS

Figure 7-3. Flow to multiple wells (adapted from John Wiley and Sons)
for the wells shown in figure 7-3 would be

\[ \varphi(x, y) = h(x, y) = h_1(x, y) + h_2(x, y) \]

\[ = \frac{1}{4\pi k_B} \left[ Q_1 \ln \left( \frac{r_1}{r_e} \right)^2 + (y - y_1)^2 \right] 
- Q_2 \ln \left( \frac{r_2}{r_e} \right)^2 + (y - y_2)^2 \] + constant

At a distance \( r_e \) from both wells \( h(x, y) = H \). The constant term is found to be

\[ \text{constant} = \frac{\ln \frac{r}{r_e}}{4\pi k_B} (Q_1 - Q_2) \]

Substituting the above equation into equation 7-9, the well formula for two wells becomes

\[ H - h(x, y) = \frac{1}{4\pi k_B} \left[ Q_1 \ln \left( \frac{r_1^2}{r_e} \right) - Q_2 \ln \left( \frac{r_2^2}{r_e} \right) \right] \]

e. Hydrologic Boundaries (Image Well Method). When there are hydrologic boundaries within the radius of influence of the well, equations 7-3 and 7-7 are no longer valid. Examples of boundaries are:

1. A stream or river which can be idealized as a line source of equal potential.
2. A rock bluff line at the edge of an alluvial fill valley which can be idealized as an impervious boundary.

The superposition of solutions (equation 7-9) can be used to analyze the flow near a boundary by introducing an artificial device called an image well. An image well is identical to the actual well and located symmetrically on the opposite side of the boundary. The superimposed effect of the real and image well for an infinite well is identical to the influence of the real well and boundary. If the real well is a pumping well then a recharging image well is used to represent boundaries such as rivers and a pumping image well is used to represent an impervious barrier. For either case, the absolute value of the flow \( Q \) for the image well is equal to that of the real well. For
example, the head distribution created by a discharging well in the vicinity of a river is identical to that created by the combined influence of a recharge and discharge well (see figure 7-4). The head distribution created by the discharge well in an infinite confined aquifer is given by

\[ h_R = \frac{Q}{2\pi k_B} \ln r_R + \text{constant} \]  

(7-10a)

and by the image recharge well in the infinite aquifer

\[ h_I = \frac{-Q}{2\pi k_B} \ln r_I + \text{constant} \]  

(7-10b)

By superposition, the head distribution for the true actual problem is

\[ h = h_R + h_I \]

\[ = \frac{Q}{2\pi k_B} \ln \frac{r_R}{r_I} + \text{constant} \]  

(7-11)

Note that at the river \( r_I = r_R \) and \( \ln \frac{r_I}{r_R} = 0 \) Thus, \( \text{constant} = H \), the head at the river. Substituting the constant term into equation 7-11, the formula for a single well near a recharge boundary is

\[ H - h = \frac{Q}{2\pi k_B} \ln \frac{r_I}{r_R} \]

To describe the head distribution for confined flow near an impervious boundary an image discharge well is used (figure 7-4b), By the procedure used above, \( h \) would be obtained as

\[ h = \frac{Q}{2\pi k_B} \ln \frac{r_R r_I}{r_I} + \text{constant} \]  

(7-12)
a. IMAGE WELL ANALYSIS OF DISCHARGE WELL NEAR RECHARGE BOUNDARY (RIVER)

b. IMAGE WELL ANALYSIS OF DISCHARGE WELL NEAR IMPERMEABLE BOUNDARY (ROCK BLUFF)

Figure 7-4. Application of image well method for analysis of flow near boundaries (courtesy of Illinois State Water Survey)
The head at the impervious boundary is unknown, thus additional information is needed to determine the constant. Note that when \( r_R \) and \( r_I \) are both equal to the radius of influence that \( h = H \). Thus

\[
h = \frac{Q}{2\pi k H} \ln \left( \frac{r_R r_I}{r_e^2} \right) + H \quad (7-13)
\]

The image well method can also be applied to problems involving multiple boundaries. For example, a common geologic situation involving multiple boundaries would be a discharge well pumping from an alluvial terrace located between a river and rock bluff (figure 7-5). In this case, the image well for the river would have a second image well with respect to the rock bluff, which in turn would have an image with respect to the river and so on. A similar progression of image wells would be needed for the impermeable barrier. Eventually, the location of each added-image well extends beyond its radius of influence \( r_e \) from the pumping well and has no practical influence in the solution.

7-4. Special Conditions. Although the simple well formula (equation 7-8) is often used to analyze flow problems, it describes a relatively idealized condition that is found rarely in practice. It is generally desirable to consider the effects of partial penetration of wells, sloping aquifer, and stratification of water bearing units in the analysis.

a. Partially Penetrating Wells. In deriving equations 7-3 and 7-7 it is assumed that the flow lines are horizontal at the entrance of the well. This assumption is valid only if the well completely penetrates the water bearing strata. An approximate solution for flow to a well partially penetrating a confined aquifer was developed by Muskat (1946). The head can be computed from

\[
h = C_1 - C_2 \beta \quad (7-14)
\]

where \( C_1 \) and \( C_2 \) are constants to be determined from boundary conditions and \( \beta \) is a function of the radius from the well (Warriner and Banks 1977). The expression for \( \beta \) given by Muskat (1946) was based on simplifying assumptions. Duncan (1963) and Banks (1965) assessed its validity from electrical analogy model studies and developed a more accurate expression for \( \beta \). The alternative empirically determined relationship for \( \beta \) developed by Duncan (1963) is given in figure 7-6. The constants \( C_1 \) and \( C_2 \) are determined
b. SECTION

c. PRIMARY IMAGE WELLS TO ACCOUNT FOR INFLUENCE OF BOUNDARIES ON REAL WELL

d. SECONDARY IMAGE WELLS TO ACCOUNT FOR INFLUENCE OF BOUNDARIES ON PRIMARY IMAGE WELLS

Figure 7-5. Multiple image wells for a two-boundary problem
Figure 7-6. Beta function curve (from Warriner and Banks)
from the boundary conditions at the well and at the radius of influence as

\[ C_1 = h_w + C_2 \beta_w \]  

\[ C_2 = \frac{H - h_w}{\beta_w - \beta_e} \]  

where

- \( h_w \) = total head at well (L)
- \( \beta_w \) = value of \( \beta \) at well radius \( r_w \) (dimensionless)
- \( H \) = total head at radius of influence \( r_e \) (L)
- \( \beta_e \) = value of \( \beta \) at radius of influence \( r_e \) (dimensionless)

The well discharge can be determined by using an empirically determined shape factor \( g \)

\[ Q = K(H - h_w)gB \]  

with

\[ g = \frac{2\pi}{\ln \frac{r_e}{B}} \frac{\beta_{4d} - \beta_e}{\beta_w - \beta_e} \]  

where

- \( \beta_{4d} \) = value of \( \beta \) at \( r = 4B \)
- \( B \) = aquifer thickness

b. Flows to Groups of Partially Penetrating Wells. An empirical method developed by Warriner and Banks (1977) provides a means to modify the relationship obtained by superimposing solutions for individual fully penetrating wells for the effects of partial penetration. First, the head at each well is computed from the assumption that they fully penetrate the aquifer:

\[ h_j = c + \frac{1}{2\pi k B} \sum_{i=1}^{N} Q_i \ln \frac{r_{ij}}{a} \]  

with \( r_{jj} = r_{wj} \)
where

\[ h_j = \text{head at well } j \, (L) \]
\[ c = \text{constant of integration} \, (L) \]
\[ Q_i = \text{discharge from well } i \, (L^3/T) \]
\[ k = \text{coefficient of permeability} \, (L/T) \]
\[ B = \text{aquifer thickness} \, (L) \]
\[ a = \text{constant} \, (L) \]
\[ r_{ij} = \text{distance between well } i \text{ and well } j \, (L) \]
\[ r_{wj} = \text{radius of well } j \, (L) \]
\[ N = \text{number of wells in group} \]

In addition, the head at a point on the source boundary is given by:

\[
H = c + \frac{1}{2\pi kB} \sum_{i=1}^{N} Q_i \ln \frac{r_{is}}{a} \tag{7-18}
\]

where \( H \) is the head at the source and \( r_{is} \) is the distance between well \( i \) and the source. The drawdown at each well is computed from combining equations 7-17 and 7-18

\[
\Delta h_j = H - h_j = \sum_{i=1}^{N} \frac{Q_i}{2\pi kB} \ln r_{is} \frac{r_{ij}}{r_{wj}} \tag{7-19}
\]

with \( r_{jj} = r_{wj} \).

Equation 7-19 gives the drawdown for each well within a group of fully penetrating wells. The values of \( Q_i \) required to cause the drawdown \( \Delta h_j \) can be determined by solving the system of \( N \) equations (7-19) for the \( N \) unknowns \( Q_i \). As for the single well, a shape factor can be defined as:

\[
S_j = \frac{Q_i}{k \Delta h_j B} \tag{7-20}
\]

7-15
where \( g_i \) is the shape factor for each well within a group. This shape factor can be corrected to account for partial penetration by

\[
g'_i = g_i \frac{r_{is}}{r_{iw}} \left( \frac{\beta_{wd} - \beta_e}{\beta_w - \beta_e} \right)
\]

(7-21)

By replacing \( g_i \) in equation 7-16 with \( g'_i \) the flow from the well group is given as

\[
Q_{total} = \sum_{i=1}^{N} k_B g'_i \Delta h_i
\]

(7-22)

The computations required to evaluate equations 7-19 through 7-22 are straightforward though they are time consuming for large well groups. Warriner and Banks (1977) provide a FORTRAN code to compute discharge and drawdowns for partially penetrating well groups within an arbitrarily shaped source boundary.

c. Wells in Sloping Aquifer. If the regional potentiometric surface has a significant slope, the effect of superimposing the initial regional gradient on the well drawdowns must be considered. For example, when pumping from floodplain locations, the existing piezometric gradient from upland areas to the river may be as great as those caused by pumping from the well. The significant parameters for confined flow to a single well are shown in figure 7-7. At a large distance from the well, the regional flow net would not be affected. All flow into the well would be contained within the stream lines separated by the dimension \( f \). Thus by Darcy's law for one-dimensional flow

\[
Q = -k \frac{dh_1}{dx} B_f
\]

(7-23)

where

\[
Q = \text{discharge from well} \ (L^3/T) \\
k = \text{permeability} \ (L/T) \\
h_1 = \text{total head} \ (L) \text{ for regional flow alone}
\]
x = coordinate selected to be parallel to initial regional flow (L)

B = aquifer thickness (L)

f = width of flow lines enclosing all flow to well (L)

Figure 7-7. Superposition of well drawdown on regional gradient (courtesy of International Institute for Land Reclamation and Improvement^{199})
The corresponding differential equation for the well would be

\[ \frac{dh_2}{dr} = \frac{Q}{2\pi kB} \frac{1}{r} \]  

(7-24)

where

\[ h_2 = \text{total head due to flow to well} \]

\[ r = \sqrt{x^2 + y^2} \]

At a distance \( X_e \), downgradient from the well, a groundwater divide develops (culmination point) at which

\[ \frac{dh_2}{dr} = -\frac{dh_1}{dx} \]  

(7-25)

In view of equations 7-23 and 7-24

\[ \frac{Q}{2\pi kB} \frac{1}{X_e} = \frac{Q}{kBf} \]  

(7-26)

or

\[ X_e = \frac{f}{2\pi} \]

By substitution of equation 7-26 into equation 7-23

\[ Q = 2\pi kX_e B \frac{dh_1}{dx} \]  

(7-27)

By integrating equations 7-24 and 7-27

\[ h_1 = \frac{Q}{2\pi kX_e B} X + c_1 \]  

(7-28)

\[ h_2 = \frac{Q}{2\pi kB} \ln r + c_2 \]
and superimposing the effects

\[ h(x,y) = h_1 + h_2 = \frac{QX}{2\pi k B X_e} + \frac{Q}{4\pi k B} \ln \left( x^2 + y^2 \right) + \text{constant} \]  

(7-29)

The distance \( X_e \) can be removed from the expression by substitution of equation 7-27

\[ h(x,y) = i x + \frac{Q}{4\pi k B} \ln \left( x^2 + y^2 \right) + \text{constant} \]  

(7-30)

where \( i = \frac{dh_1}{dx} \) the regional slope of the aquifer.

For conditions of unconfined flow, the regional gradient would be defined by a parabola

\[ h_1^2 = \frac{2Q}{K_f} x + \text{constant} \]

which when combined with the well equations for unconfined flow gives

\[ h^2(x,y) = \frac{Q}{\pi k B} \left[ \frac{x}{X_e} + \frac{1}{2} \ln \left( x^2 + y^2 \right) \right] + \text{constant} \]  

(7-31)

d. Layered Aquifers. Natural soils often occur in layers and a well may penetrate units having different permeabilities. If flow to the well is horizontal, the simple well equations can be used by assigning an average value of permeability given by

\[ k_{\text{avg}} = \sum_{m=1}^{N} \frac{k_m d_m}{d} \]  

(7-32)

where

\( k_m = \) horizontal permeability of layer \( m \)

\( d_m = \) thickness of layer \( m \)

\( d = \) total thickness of layers
Note that the permeability determined from a field pumping test is an average of all units penetrated by the pumping well. A case where vertical flow can be important is shown in figure 7-8. The discharging well is pumping from a permeable unit overlain by a less permeable unit through which significant vertical flow can occur. The flow to the well is given by

\[ H - h = \frac{Q}{2\pi k B^2} K \left( \frac{r}{L} \right) \]  

**(7-33)**

where

- \( H \) = original total head (L)
- \( h \) = total head at distance \( r \) from well at steady state condition (L)
- \( Q \) = discharge rate (L³/T)
- \( L = \sqrt{kBC} \) (leakage factor) (L)
- \( B \) = thickness of aquifer
- \( C = B'/k' \) (L)
- \( B' \) = thickness of overlying low permeability unit (L)
7-5. Nonsteady State Flow. Nonsteady state flow may arise in several ways. When pumping is started, time is required to establish a virtually steady state condition. Flow during this period must be assumed to be nonsteady state. If pumping occurs intermittently, a steady state condition may not be established. Also, if large fluctuations occur at the source, potential steady state flow conditions are not maintained. The steady state condition can be viewed as the end condition that is reached after pumping for a long time period. In the design of a well system for seepage control, it is generally adequate to consider only the steady state condition. However, the determination of coefficient of permeability from test data often requires analysis based on nonsteady state condition. The duration of many well tests is too short to

Table 7-1. Values of $K_o r/L$ for Selected Values of $r/L$ to Evaluation Equation 7-33

<table>
<thead>
<tr>
<th>$\frac{r}{L}$</th>
<th>$f = 10^{-2}$</th>
<th>$f = 10^{-1}$</th>
<th>$f = 1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>4.721</td>
<td>2.427</td>
<td>0.421</td>
</tr>
<tr>
<td>2.0</td>
<td>4.028</td>
<td>1.753</td>
<td>0.114</td>
</tr>
<tr>
<td>3.0</td>
<td>3.623</td>
<td>1.372</td>
<td>0.035</td>
</tr>
<tr>
<td>4.0</td>
<td>3.336</td>
<td>1.114</td>
<td>0.011</td>
</tr>
<tr>
<td>5.0</td>
<td>3.114</td>
<td>0.924</td>
<td>0.004</td>
</tr>
<tr>
<td>6.0</td>
<td>2.933</td>
<td>0.777</td>
<td>0.000</td>
</tr>
<tr>
<td>7.0</td>
<td>2.780</td>
<td>0.660</td>
<td>0.000</td>
</tr>
<tr>
<td>8.0</td>
<td>2.647</td>
<td>0.565</td>
<td>0.000</td>
</tr>
<tr>
<td>9.0</td>
<td>2.531</td>
<td>0.487</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Example: $\frac{r}{L} = 0.5$, $f = 10^{-1}$, $K(0.5) = 0.924$

(a) Prepared from more extensive tables presented by Kruseman and De Ridder (1970).
reliably establish the steady state condition. Also, in practice, hydrologic boundaries may be present within the steady state radius of influence. In either case the use of the steady state flow equations could lead to substantial error in determining the permeability.

a. Nonsteady State Confined Flow. Theis (1935) developed the following relationship for nonsteady state flow in a confined aquifer (Davis and DeWeist 1966):

\[ H - h = \frac{Q}{4\pi kb} W(u) \]  

(7-34)

where

\[ Q = \text{constant discharge rate (L}^3/\text{T)} \]
\[ k = \text{permeability (L/T)} \]
\[ B = \text{thickness of aquifer (L)} \]
\[ W(u) = \text{function given in table 7-2} \]

\[ W(u) = -0.5772 - \ln u + u - \frac{u^2}{2 \times 2!} + \frac{u^3}{3 \times 3!} - \frac{u^4}{4 \times 4!} + \ldots \]

The parameter \( u \) is given by

\[ u = \frac{r^2 S}{4kBt} \]  

(7-35)

where

\[ r = \text{radius from well (L)} \]
\[ S = \text{storage coefficient (dimensionless)} \]
\[ t = \text{time from start of pumping (T)} \]

The storage coefficient \( S \) represents the amount of water removed from storage as a result of consolidation of the aquifer and expansion of water in response to the decline in head. Physically \( S \) is given by

\[ S = \rho g B (\alpha + n \beta) \]  

(7-36)
Table 7-2. Values of $W(u)$ for Selected Values of $1/u$

to Evaluate Equation 7-34\(^{(a)}\)

<table>
<thead>
<tr>
<th>$\frac{1}{f\cdot u}$</th>
<th>$f = 1.0$</th>
<th>$f = 2.0$</th>
<th>$f = 8.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-1}$</td>
<td>0.000</td>
<td>0.001</td>
<td>0.146</td>
</tr>
<tr>
<td>1</td>
<td>0.219</td>
<td>0.600</td>
<td>1.623</td>
</tr>
<tr>
<td>10</td>
<td>1.823</td>
<td>2.468</td>
<td>3.817</td>
</tr>
<tr>
<td>$10^2$</td>
<td>4.034</td>
<td>4.726</td>
<td>6.109</td>
</tr>
<tr>
<td>$10^3$</td>
<td>6.332</td>
<td>7.024</td>
<td>8.410</td>
</tr>
<tr>
<td>$10^4$</td>
<td>8.633</td>
<td>9.326</td>
<td>10.71</td>
</tr>
<tr>
<td>$10^5$</td>
<td>10.94</td>
<td>11.63</td>
<td>13.02</td>
</tr>
<tr>
<td>$10^6$</td>
<td>13.24</td>
<td>13.93</td>
<td>15.32</td>
</tr>
<tr>
<td>$10^7$</td>
<td>15.54</td>
<td>16.23</td>
<td>17.62</td>
</tr>
<tr>
<td>$10^8$</td>
<td>17.84</td>
<td>18.54</td>
<td>19.92</td>
</tr>
<tr>
<td>$10^9$</td>
<td>20.15</td>
<td>20.84</td>
<td>22.22</td>
</tr>
<tr>
<td>$10^{10}$</td>
<td>22.45</td>
<td>23.14</td>
<td>24.53</td>
</tr>
<tr>
<td>$10^{11}$</td>
<td>24.75</td>
<td>25.44</td>
<td>26.83</td>
</tr>
<tr>
<td>$10^{12}$</td>
<td>27.05</td>
<td>27.75</td>
<td>29.13</td>
</tr>
<tr>
<td>$10^{13}$</td>
<td>29.36</td>
<td>30.05</td>
<td>31.44</td>
</tr>
<tr>
<td>$10^{14}$</td>
<td>31.66</td>
<td>32.35</td>
<td>33.74</td>
</tr>
</tbody>
</table>

Example: $u = 0.005$, $\frac{1}{u} = 200$, $f = 2$, $W(u) = 4.726$

\(^{(a)}\) Prepared by WES.
where
\[ \rho = \text{mass density fluid (m/L}^3) \]
\[ g = \text{acceleration of gravity (L/T}^2) \]
\[ B = \text{thickness of aquifer} \]
\[ \alpha = \text{bulk compressibility of aquifer (LT}^2/M) \]
\[ n = \text{porosity (dimensionless)} \]
\[ \beta = \text{bulk compressibility of fluid (LT}^2/M) \]

The determination of the aquifer properties \( k_B \) and \( S \) from equation 7-34 requires a complete drawdown versus time history for each observation piezometer. The Theis method for data analysis is based on the logarithmic representation of equations 7-34 and 7-35

\[
\log (H - h) = \log [W(u)] + \log \left( \frac{Q}{4\pi k_B} \right)
\]
\[
\log \left( \frac{r^2}{t} \right) = \log (u) + \log \left( \frac{S}{r k_B} \right)
\]

From the equations above it is seen that if \( Q \) is constant that \( \log (H - h) \) varies with \( \log (r^2/t) \) in the same way as \( \log [W(u)] \) varies with \( \log (u) \) regardless of the units used. Therefore, it should be possible to superimpose the data curve on the theoretical curve because the two curves are offset from each other only by the constant terms \( \log Q/4\pi k_B \) and \( \log S/4k_B \). By determining the value of the offsets from the superimposed curves, \( k_B \) and \( S \) can be determined. The computation consists of the following steps:

1. A plot is made of \( W(u) \) (log scale) versus \( u \) (log scale). This plot is referred to as the type curve.

2. For each observation well, a plot is made of drawdown \( H - h \) (log scale) versus \( \frac{r^2}{t} \) (log scale).

3. Superimpose the test data over the type curve in such a way that the drawdown data best fit the type curve (figure 7-9). The coordinate axes of the two curves should be kept parallel.

4. Determine the values \( W(u) \), \( u \), \( H - h \), and \( \frac{r^2}{t} \) from an arbitrarily chosen matching point on the two curves.
(5) Compute the value of $k_B$ from equation 7-34 using the matching point value of $H - h$ and $W(u)$. Compute the value of $S$ from equation 7-35 using the matching point values of $u$ and $r^2/t$ combined with the previously computed value of $k_B$. The above procedure is carried out for each observation well. Ideally, the computed values of $k_B$ and $S$ should be the same for all observation wells. Differences in the computed values may be caused by geologic variations in the aquifer and hydrologic boundaries not accounted for in the analysis.

b. Simple Method for Coefficient Determination (Jacob's Method). Jacob (1950) introduced a simplification to the determination of $k_B$ and $S$ by noting that for small values of $u$ (small $r$ and/or large $t$) equation 7-34 reduces to (Davis and DeWeist 1966)
Equation 7-37 can be written in a form convenient for graphical solution by substituting equation 7-35 and writing in terms of base 10 logarithms:

\[
H - h = \frac{Q}{4\pi k_B} \left( \ln \frac{1}{u} - \ln 1.78 \right); \quad u \leq 0.01
\]  

(7-37)

From equation 7-38 it is seen that the relationship between drawdown \( H - h \) and time \( t \) for a particular observation piezometer \( (r = \text{constant}) \) can be represented as a straight line on a plot of \( H - h \) versus \( \log t \) (figure 7-10). The slope of the line is equal to \( \frac{2.30Q}{4\pi t} \). Also, the time, \( t_0 \), corresponding to \( H - h = 0 \) gives

\[
\frac{2.25T t_0}{r^2 S} = 1
\]

which can be used to determine \( S \). An alternative analysis consists of plotting \( H - h \) versus \( \log r \). The following relationship can be obtained by rearranging the term in equation 7-38.

\[
H - h = \frac{2.30Q}{4\pi k_B} \log \frac{2.25T t}{r^2 S} - \frac{2.30Q}{2\pi k_B} \log r
\]  

(7-39)

Equation 7-39 defines a straight line on a plot of \( H - h \) versus \( r \) (figure 7-10b). The slope of the line is \( -\frac{2.30Q}{2\pi k_B} \) and can be used to determine \( k_B \). The line intersects the \( H - h = 0 \) axis at \( r_0 \). This intercept can be used to determine \( S \) from

\[
\frac{2.25k_B t}{r_0^2 S} = 1
\]

Note that \( r_0 \) represents the radius of influence for the well at time equals \( t \). Thus the radius of influence for the steady state condition \( r_0 \) is equal to \( r_0 \) as \( t \) tends to infinity. This implies that the radius of influence
a. One observation well

b. Simultaneous observations

Figure 7-10. Use of Jacob approximation for nonsteady state flow (courtesy of John Wiley and Sons)
expands indefinitely and cannot be defined. However, the value of \( r_e \) selected has a relatively small influence on computed drawdowns near the well and equation 7-39 can be used to determine reasonable values for \( r_e \).

c. Nonsteady Unconfined Flow with Vertical Gravity Drainage (Delayed Yield). Initial response (generally after first few minutes of pumping) is given by (Kruseman and DeRidder 1970)

\[
H - h = \frac{Q}{4\pi k B} W(u_A, r/B) \tag{7-40}
\]

where

\[
u_A = \frac{r^2 S_A}{4kBt}
\]

\( S_A \) = storage coefficient for instantaneous release of water from storage

\( W(u_A, r/B) \) = Boulton well function (figure 7-11a)

\( r/B \) = formation constant to be determined from piping test data

Later time response is given by

\[
H - h = \frac{Q}{4\pi k B} W(u_y, r/B) \tag{7-41}
\]

where

\[
u_y = \frac{r^2 S_y}{4kBt}
\]

\( S_y \) = specific yield

\( W(u_y, r/B) \) = delayed yield well function

The application of equations 7-40 and 7-41 through use of a type-curve is similar to that of equation 7-34. The following should also be noted:

(1) Type curves for several values of \( r/B \) should be plotted. The curve giving the best fit to the initial time-drawdown data is used to estimate \( r/B \).
Family of Boulton type curves: $W(U_y, r/B)$ versus $1/u_y$ and $W(U_x, r/B)$ versus $1/u_x$ for different values of $r/B$.

b. Boulton's delay index curve

Figure 7-11. Type curves for Boulton's analysis of nonsteady unconfined flow with delayed yield (courtesy of International Institute for Land Reclamation and Improvement)
(2) The time-drawdown data overlay may be moved to obtain the best fit for the latter time-drawdown data. Both initial time and latter time fits should give the same value of \( r/B \) and \( k_B \).

(3) Eventually, the effects of vertical gravity drainage become negligible and the latter time curve merges with the Theis curve. The time-coordinate where the two curves merge is determined from Boulton's delay-index curve (figure 7-11b).

(4) A number of type-curve solutions to the problem of nonsteady unconfined flow to wells have been developed (Fetter 1980). For example, Neuman (1975) presented a type-curve method similar to Boulton's that accounts for anisotropy of the aquifer.

d. Nonsteady Confined Flow with Vertical Drainage Through Confining Layer (Leaky Aquifer). The leaky aquifer equation for nonsteady flow is based on the assumptions that flow to the well is horizontal and vertical flow is restricted to seepage through the confining layer. These assumptions are identical to those made for the steady state case described by equation 7-33. The drawdown is given by

\[
H - h = \frac{Q}{4\pi k B} W\left(u, \frac{r}{L}\right) \tag{7-42}
\]

where

\[
\begin{align*}
\frac{r^2 S}{u} & = 4kBt \\
r & = \text{radius from well (L)} \\
S & = \text{storage coefficient (dimensionless)} \\
k & = \text{permeability (L/T)} \\
t & = \text{time from start of pumping (T)} \\
L & = \text{leakage factor (L)} = \frac{k B B'}{k'} \\
B & = \text{thickness of aquifer (L)} \\
B' & = \text{thickness of confining unit (L)} \\
k' & = \text{permeability of confining unit (L/T)} \\
W(u, r/L) & = \text{well function given in figure 7-12}
\end{align*}
\]
Figure 7-12. Type curve for nonsteady flow in a semiconfined aquifer (courtesy of John Wiley and Sons)
The application of the type curve method for the leaky aquifer problem is similar to the application to the delayed yield problem. The time-drawdown data are matched to the standard type curve with the curve giving the best fit being used to estimate \( r/B \).

e. Nonsteady Unconfined Flow with Little Vertical Drainage. If the delayed response component of the drawdown is small, the Theis equation (equation 7-34) can be used to analyze the flow by inserting a "corrected" drawdown into the flow equation. The corrected drawdown is given by

\[
(H - h)' = (H - h) - \left[ \frac{(h - H)^2}{2H} \right]
\]  

(7-43)

f. Nonsteady Flow with Hydrologic Boundaries. The method of superposition presented for steady-state flow problems (equation 7-9) is applicable to nonsteady flow problems. Therefore, the image well method can be used to investigate the effects of hydrologic boundaries. For example, the image well analysis for a discharging well near a river (recharge boundary) is (Davis and Dewiest 1966)

\[
H - h = \frac{Q}{4\pi k B} \left[ W(u_R) - W(u_I) \right]
\]  

(7-44)

where

\[
\begin{align*}
  u_R &= \frac{r^2 S}{4k B t} \\
  u_I &= \frac{r_I^2 S}{4k B t} \\
  r_R &= \text{radius from real well (L)} \\
  r_I &= \text{radius from image well (L)} \\
  k &= \text{coefficient of permeability (L/T)} \\
  S &= \text{storage coefficient (dimensionless)} \\
  B &= \text{aquifer thickness (L)} \\
  t &= \text{time from start of pumping (T)}
\end{align*}
\]
Note, then, when the function $W(u)$ can be replaced with a logarithmic approximation, as in the Jacob's method (equation 7-37), equation 7-44 can be approximated as

$$H - h = \frac{Q}{4\pi kB} \ln \frac{r_I^2}{r_R^2} - \ln \frac{r_I^2}{r_R^2} = \frac{Q}{2\pi kB} \ln \frac{r_I}{r_R} \quad (7-45)$$

From equation 7-45 it is seen that as $u$ becomes small, flow becomes virtually steady state (compare equation 7-45 with the steady state case, equation 7-11). Thus the presence of a recharge boundary in an aquifer tends to shorten the time needed to reach steady state (Davis and Dewiest 1966).
CHAPTER 8
SEEPAGE CONTROL IN EMBANKMENTS

8-1. General. All earth and rock-fill dams are subject to seepage through the embankment, foundation, and abutments. Seepage control is necessary to prevent excessive uplift pressures, instability of the downstream slope, piping through the embankment and/or foundation, and erosion of material by migration into open joints in the foundation and abutments. The purpose of the project, i.e., long-term storage, flood control, etc., may impose limitations on the allowable quantity of seepage.

8-2. Methods for Seepage Control. The three methods for seepage control in embankments are flat slopes without drains, embankment zonation, and vertical (or inclined) and horizontal drains.

8-3. Flat Slopes Without Drains. For some dams constructed with impervious soils having flat embankment slopes and infrequent, short duration, high reservoir levels, the phreatic surface may be contained well within the downstream slope and escape gradients may be sufficiently low to prevent piping failure. For these dams, when it can be assured that variability in the characteristics of borrow materials will not result in adverse stratification in the embankment; no vertical or horizontal drains are required to control seepage through the embankment. A horizontal drain may still be required for control of underseepage (see Chapter 9). Examples of dams constructed with flat slopes without vertical or horizontal drains are Aquilla Dam, Aubrey Dam, and Lakeview Dam (U. S. Army Engineer District, Fort Worth 1976a, 1976b, 1980). Figure 8-1 shows the analysis of through embankment seepage for Aubrey Dam, Texas (now called Ray Roberts Dam). As shown in figure 8-1a, this is a zoned embankment with relatively flat slopes due to a weak stratum in the foundation. The slopes could be steepened from IV:10.6H to IV:8H if the weak foundation gains shear strength due to consolidation during construction (the dam is scheduled for completion in 1988). A 3-ft-thick horizontal drainage blanket and collector system will be provided under the downstream embankment from sta 136+00 to sta 142+60 to control any seepage through the foundation. For the analysis of through embankment seepage, shown in figure 8-1b, the steady state phreatic surface was developed graphically for the conservation pool by considering a homogeneous nonisotropic embankment and an impervious foundation. Since the escape seepage gradients were computed to be less than 0.3 to 0.4 (see paragraph 4.9b), it was concluded that no vertical or horizontal drains were required.

8-4. Zoning Embankments.

a. General. Embankments are zoned to use as much material as possible from required excavation and from borrow areas with the shortest haul distances and the least wastage and at the same time maintain stability and control seepage. The different zones of an embankment are shown in figure 8-2. For most effective control of through seepage and seepage during reservoir drawdown, the permeability should progressively increase from the core out toward each slope as shown in figure 8-2 (EM 1110-2-2300).
Figure 8-1. Analysis of through embankment seepage for Aubrey Dam, Texas (from U. S. Army Engineer District, Fort. Worth)
b. Impervious Zone or Core. The purpose of the core is to minimize seepage losses through the embankment. As a general rule, sufficient impervious material is available to result in small seepage losses through the embankment. Therefore, the quantity of seepage passing through the foundation and abutments may be more significant than the quantity passing through the core. Important material properties of the core are permeability, erosion resistance, and cracking resistance. A core material of very low permeability may be required when the reservoir is used for long-term storage. A core material of medium permeability may be utilized when the reservoir is used for flood control. The erosion resistance of core material is important in evaluating piping potential (Arulanandan and Perry 1983). The tensile strength of the core material is important in evaluating the cracking resistance (Al-Hussaini and Townsend 1974). In general, the base of the core or the cut-off trench should be equal to or greater than a quarter of the maximum difference between reservoir and tailwater elevations (U. S. Army Engineer District, Mobile 1976, EM 1110-2-2300). A core top width of 10 ft is considered to be the minimum width on which earth-moving and compaction equipment can operate. The maximum core width is controlled by stability and by availability of material. The top of the core should be above the maximum reservoir elevation.
but below the bottom of the frost zone, a vertical core located near the center of the dam is performed over an inclined upstream core because the former provides higher contact pressure between the core and foundation to prevent leakage, greater stability under earthquake loading (Sherad 1966, 1967), and better access for remedial seepage control. An inclined upstream core allows the downstream portion of the embankment to be placed first and the core later and reduces the possibility of hydraulic fracturing (Nobari, Lee, and Duncan 1973).

c. **Filter Zones.** Filters may be required in various locations in earth dams such as vertical (or inclined) and horizontal drains within the downstream section of the embankment as shown in figure 8-2, around outlet conduits passing under the downstream portion of the embankment, under concrete structures such as stilling basins, around relief wells, beneath riprap where drawdown may occur, and between the embankment and abutment. Important properties of the filter material are gradation, compacted density, and permeability. Filters are designed to permit free passage of water and prevent migration of fines through the filter as discussed in Appendix D. The average in-place relative density of the filter should be at least 85 percent and no portion of the filter should have a relative density less than 80 percent (EM 1110-2-2300). This requirement applies to vertical (or inclined) and horizontal drains and filters under concrete structures but not to bedding layers under riprap. Special care must be taken to assure that compaction does not degrade the filter material (by grain breakage and/or segregation) and reduce its permeability. When the filter material is sand or contains significant portions of sand sizes, the material should be maintained in as saturated a condition as possible during compaction to prevent bulking. The discharge capacity of the filter zones should be determined in dimensioning the filters (Cedergren 1977). The filter material should pass the 3-in. screen for minimizing particle segregation and bridging during placement. As discussed previously in Chapter 2 (see figure 2-12), the permeability of sands and gravels varies significantly with the amount and type of fines (material smaller than the No. 200 sieve) present. Also, the amount and type of fines present influence the capacity of a filter to self-heal by collapsing any cracks within the filter (see figure 8-3). Therefore, the maximum percent fines and type (silt, clay, etc.) to be allowed in the filter of an earth dam must be shown to be sufficiently pervious by laboratory filter tests\(^{(1)}\) and self healing by collapse tests (Vaughn 1978). If vibration is present, such as in the vicinity of a stilling basin or powerhouse, the laboratory filter tests should be conducted with vibration effects. If the base material to be protected is dispersive, large-scale box filter tests will be required (McDaniel and Decker 1979, Bordeaux and Imaizumi 1977, and Logani and Lhez 1979). The procedure to use in identifying dispersive clays is given in EM 1110-2-1906. Generally, two or more filter zones, each with a uniform or narrow gradation (sand, pea gravel, etc.) are preferable to a single well-graded filter zone which often becomes segregated during processing, stockpiling, and placement. Care must

\(^{(1)}\) Laboratory filter tests are not a routine laboratory test. Standard testing procedures have not been developed. The conduct of laboratory filter tests should be under the direction of a specialist and should be carried out in a research laboratory.
a. Self-healing (by collapse) of filter

b. Laboratory test for ability of filter material to self-heal (by collapse)

Figure 8-3. Self-healing (by collapse) of crack within a filter downstream of a core (courtesy of American Society of Civil Engineers)

be taken during construction to prevent reduction in permeability of the filter by intrusion of fines carried by surface runoff, spillage by compaction equipment, or degradation during compaction. Also, care must be taken to prevent coarse material from rolling down the surface of the filter and collecting between the core and filter (or between filter zones if two or more filters are used) forming a "tube" (in cross section) of more permeable material through which core (or filter) material could be lost by piping.
d. **Transition Zones.** The purpose of transition zones is to separate zones of different permeability and compressibility within the embankment, to prevent core material from being drawn into the upstream shell during rapid drawdown of the reservoir, to provide a source to feed material into a crack in the core and preventing piping (see figure 8-4). Important material properties of the transition material are angularity of particles (upstream of core, rounded particles and better for feeding material into cracks and preventing piping), gradation, permeability, and compressibility. Transition zones may be located both upstream and downstream of the core and generally have a width >10 ft. Wide transition zones between the filter and the downstream shell will control the rate of flow through a crack in the core and extending through the filter in the event that self-healing (by collapse) of the filter does not occur. Transition zones (and filter zones) should be widened near abutments where tension zones may induce cracking.

![Figure 8-4. Crack stopping transition upstream of a core (prepared by WES)](image)

e. **Random Zones.** The purpose of random zones is to utilize required excavation. Random zones are assumed to have the properties of the least desirable material in the excavation. Random zones may be located either upstream or downstream of the core. For most effective control of through seepage and seepage during reservoir drawdown, the more pervious material should be routed to the outer portions of the embankment.

f. **Cuter Zones or Shells.** The purpose of the outer zones or shells is to permit steeper embankment slopes. Important material properties of the shell material are durability (soundness) of rock, gradation (well-graded), and permeability (free-draining). The upstream shell affords stability
against end of construction, rapid drawdown, and earthquake loading. The downstream shell acts as a drain and controls the line of seepage and provides stability under high reservoir heads. When suitable materials are not available for pervious downstream shells, control of seepage through the embankment is provided by vertical (or inclined) and horizontal drains.

q. **Upstream Drawdown Blanket.** The purpose of the upstream drawdown blanket is to provide stability of the upstream slope during rapid drawdown of the reservoir. Important material properties of the upstream drawdown blanket are durability (soundness) of rock and permeability (free-draining). Figure 8-5 shows the improvement in the factor of safety resulting from the

![Diagram of factor of safety improvement](image)

---

(a) S.F. = 1.08

(b) S.F. = 1.48

1: RELATIVELY IMPERVIOUS

2: MODERATELY PERVERSIVE, NOT FREE DRAINING

3: HIGHLY PERVERSIVE, FREE DRAINING

Figure 8-5. Improvement in factor of safety resulting from the upstream drawdown blanket (courtesy of John Wiley and Sons)
upstream drawdown blanket (Cedergren 1977). The required minimum permeability of the upstream drawdown blanket can be calculated from (Cedergren 1977):

\[
k_{\text{min}} = \frac{v_{\text{dd}} L n_e}{h \sin \alpha}
\]  

where

- \(k_{\text{min}}\) = minimum permeability of the upstream drawdown blanket
- \(v_{\text{dd}}\) = velocity of drawdown of the water surface in the reservoir
- \(L\) = defined in figure 8-6
- \(n_e\) = effective porosity of the upstream drawdown blanket
- \(h\) = defined in figure 8-6
- \(\alpha\) = angle between the median flow line in the upstream drawdown blanket and the horizontal

The ratio \(h/L\) should not exceed 10.

Figure 8-6. Computation of required minimum permeability of the upstream drawdown blanket (courtesy of John Wiley and Sons).

8-5. Vertical (or Inclined) and Horizontal Drains.

a. Need. As stated previously, vertical (or inclined) and horizontal drains may be required to control seepage through the embankment by preventing material eroded through a crack in the core from washing into the downstream shell by seepage water under reservoir head. Also, because of the often variable characteristics of borrow materials, it is frequently advisable to provide vertical (or inclined) and horizontal drains within the downstream section of the embankment, as shown in figure 8-7, to ensure satisfactory seepage control. For a stratified soil, the vertical permeability is
a. HORIZONTAL DRAIN ONLY

b. INCLINED AND HORIZONTAL DRAINS

Figure 8-7. Use of inclined and horizontal drains to ensure seepage control against variable characteristics of borrow materials (courtesy of John Wiley and Sons).

controlled by the least permeable layer. Therefore, the horizontal permeability is always greater than the vertical permeability. Compacted soils in earth dams are stratified due to variability in the characteristics of borrow materials and the tendency for soil particles to align horizontally during compaction. The ratio of vertical to horizontal permeability may range from 2 to 10 or greater. For stratified soils, as shown in figure 8-8, a horizontal drainage blanket is not sufficient to prevent the downstream slope from becoming saturated and susceptible to piping and/or slope failure. However, when a properly designed and constructed inclined drain and horizontal drain is used, as shown in figure 8-8, complete control is provided over seepage through the embankment.

b. Filter Requirements. Vertical (or inclined) and horizontal drains should be designed as filters (see Appendix D). If crushed rock is used for the drain material (see paragraph 2-2.g), material to be protected is dispersive, or material to be protected contains cracks, filter tests will be
Figure 8-8. Effect of anisotropy of permeability on seepage through earth dam with and without an inclined drain (courtesy of John Wiley and Sons$^{155}$)
Well-graded materials are internally unstable and should not be used as filters when $C_u > 20$.\(^{(1)}\)

c. **Discharge Requirements.** Vertical (or inclined) and horizontal drains must have sufficient discharge capacities to remove seepage quickly without inducing high seepage forces or hydrostatic pressures (Cedergren 1977). When drains are designed and constructed with ample discharge capacity, the line of seepage will not rise above the drain zone. Since drains are small compared to the overall dimensions of the earth dam, it is difficult to construct accurate flow nets within the drains themselves. The total quantity of seepage from all sources that must discharge through the drain should be evaluated from a flow net analysis in which it is assumed that the drains have an infinite permeability. Figure 8-9 shows an example of the design procedure for inclined and horizontal drains to assure adequate drain capacity (Cedergren 1977). The probable rate of discharge through the dam and foundation is estimated from composite flow nets (see figures 4-13 and 4-14). For the example shown in figure 8-9, the seepage through the dam $Q_1 = 2$ cu ft/day and the seepage through the foundation $Q_2 = 10$ cu ft/day. Therefore, the inclined drain must be capable of discharging $Q_1 = 2$ cu ft/day and the horizontal drain must be capable of discharging $Q_1 + Q_2 = 12$ cu ft/day. These are discharge rates per running foot of dam and drain. Assuming the inclined drain was designed with a width of 12 ft to permit its placement with normal earth-moving equipment, the cross-sectional area normal to the direction of flow within the inclined drain $A_c = 11$ sq ft (see figure 8-9b) and its required minimum permeability may be found from Darcy's law:

$$k = \frac{Q}{1A}$$

\(^{(8-2)}\)

\(^{(1)}\) $C_u = \frac{D_{60,F}}{D_{10,F}}$

where $C_u$ = coefficient of uniformity

$D_{60,F}$ = size of filter material at 60 percent passing

$D_{10,F}$ = size of filter material at 10 percent passing
where

\[ k = \text{coefficient of permeability} \]
\[ Q = \text{quantity of discharge} \]
\[ i = \text{hydraulic gradient} \]
\[ A = \text{cross-sectional area of flow} \]

Figure 8-9. Example of design procedure for inclined and horizontal drains to assure adequate drain capacity (courtesy of John Wiley and Sons)
Substituting for $Q_1$, $l$, and $A$

\[ k_c = \frac{Q_1}{h_c \left( A_c \right)} \]

\[ k_c = \frac{2 \text{ cu ft/day}}{\left( \frac{300 \text{ ft}}{310 \text{ ft}} \right) \text{ 1 sq ft}} \quad (8-3) \]

\[ k_c = 0.2 \text{ ft/day} \]

Every filter must be permeable enough to have a reasonable reserve for higher than expected flows. The filter should have a minimum permeability after placement and compaction of at least 20 times that calculated theoretically. Therefore, the required permeability for the inclined drain is

\[ k_{c_{\text{design}}} = 20k_c \]

\[ k_{c_{\text{design}}} = 20(0.2 \text{ ft/day}) = 4 \text{ ft/day} \quad (8-4) \]

\[ k_{c_{\text{design}}} = 1.4 \times 10^{-3} \text{ cm/sec} \]

Clean, washed concrete sand is usually about this permeable. As previously stated, the horizontal drain must be capable of discharging $Q_1 + Q_2 = 12 \text{ cu ft/day}$. Since the drain is to be designed so that the line of seepage does not rise above the drain zone, the allowable maximum head in the horizontal drain can be no greater than its height. The required minimum permeability from Darcy's law is

\[ k_b = \frac{Q_1 + Q_2}{\left( \frac{h_b}{L_b} \right) A_b} \quad (8-5) \]

Substituting $A_b = h_b$ (width is one running foot of dam and drain)
To design the horizontal drain, select a drain height and calculate the required minimum permeability. Apply a factor of safety of 20 to the calculated permeability and select a drain material from available aggregates. Select a drain height of 4 ft.

\[
k_b = \frac{Q_1 + Q_2}{h_b} \left( \frac{h_b}{h_b} \right) h_b
\]

\[
k_b = \frac{(Q_1 + Q_2)L_b}{h_b^2}
\]

\[
k_b = \frac{6,600}{h_b^2}
\]

The required permeability for design is

\[
k_{b_{\text{design}}} = 20 k_b
\]

\[
k_{b_{\text{design}}} = 20 (412.5 \text{ ft/day}) = 8,250 \text{ ft/day}
\]

\[
k_{b_{\text{design}}} = 2.9 \text{ cm/sec}
\]

This permeability could be obtained by screened fine gravel (3/8-in. to 1/2-in. size) which has a permeability of about 30,000 ft/day or 10.6 cm/sec.
(Cedergren 1977). Seepage in coarse aggregate is likely to be turbulent and a reduction factor should be applied to the permeability. The hydraulic gradient in the horizontal drain is

\[ i = \frac{h_b}{L_b} = \frac{4 \text{ ft}}{550 \text{ ft}} = 0.007 \]  

(8-7)

From figure 8-10, for screened fine gravel (3/8-in. to 1/2-in. size) with \( i = 0.007 \) the reduction factor for permeability is 0.9.

\[ k = 0.9 \ (30,000 \ \text{ft/day}) \]

\[ k = 27,000 \ \text{ft/day} \]  

(8-8)

\[ k = 9.5 \ \text{cm/sec} \]

The permeability of the screened fine gravel (3/8-in. to 1/2-in. size) reduced for turbulence is greater than the required permeability for design:

\[ 27,000 \ \text{ft/day} > 8,250 \ \text{ft/day} \]

Therefore, it should be adequate when properly placed and compacted to conduct seepage water through the horizontal drain. The screened fine gravel (3/8-in. to 1/2-in. size) will be protected top and bottom with a 1-ft-thick clean washed concrete sand filter. Since the seepage from the foundation must flow across the fine filter to enter the coarse drainage layer, the fine filter must be permeable enough to allow the water to enter the coarse drainage layer freely under only a small hydraulic gradient (0.5 or less). Assume an average hydraulic gradient of 0.5 across the fine filter layer and \( Q_2 = 10 \ \text{cu ft/day} \) for the amount of water that will enter the first (left) 200 ft of the drain. From Darcy's law the required minimum permeability of the fine filter is (see equation 8-2)

\[ k = \frac{Q}{1A} \]

\[ k = \frac{10 \ \text{cu ft/day}}{0.5(200 \ \text{ft})(1 \ \text{ft})} \]

\[ k = 0.1 \ \text{ft/day} \]
Figure 8-10. Approximation for estimating reduction in permeability of narrow size-range aggregate caused by turbulent flow (courtesy of John Wiley and Sons).

Clean washed concrete sand with a permeability of 10 ft/day should allow seepage from the foundation to enter the coarse drainage layer without restriction. As stated previously, the inclined and horizontal drains used in this example would have to meet the filter requirements (see Appendix D) in addition to the discharge requirements.

d. Location and Geometry. Vertical (or inclined) drains are located adjacent to and downstream of the core as shown in figure 8-2. The top of the vertical (or inclined) drain should be above the phreatic surface for maximum reservoir elevation to prevent seepage flow above the drain. In drawing the flow net for the dam to use in selecting the height and location for the vertical or inclined drain, a conservative (high) value of anisotropy of permeability of the soil should be used in order to prevent the seepage from flowing over the top of the drain (see figure 8-11). If the dam is located where earthquake effects are likely (see paragraph 8-6), the vertical (or inclined) drain should extend the full height of the dam. The width of the vertical (or inclined) drain is controlled by the availability of materials and the discharge requirements of the drain. A vertical (or inclined) drain width of 6 ft is the practical minimum for earth-moving and compaction equipment (U. S. Army Engineer District, Kansas City 1974 and U. S. Army Engineer District, Philadelphia 1974). When filter or drain material is not available locally and must be hauled to the site at a substantial cost, a narrow (3 ft or greater) vertical drain may be constructed by excavating into the core material, backfilling, and compacting with vibratory equipment (U. S. Army Engineer District, Mobile 1965, U.S. Army Engineer District, Tulsa 1974). The width of the vertical drain must be sufficient to satisfy the discharge requirements. Horizontal drains are located under the downstream section of the dam and convey seepage from the vertical (or inclined) drain and underseepage from the...
Figure 8-11. Use of stringers or finger drains as an alternative to a continuous horizontal drain (prepared by WES)

Foundation to the toe of the dam. The thickness of the horizontal drain must be sufficient to satisfy the discharge requirements. When filter or drain material is not available locally and must be hauled to the site at substantial cost, a thin (2 ft or greater) horizontal drain has been used (U. S. Army Engineer District, Tulsa 1975 and U. S. Army Engineer District, Louisville 1974). Stringers or finger drains may be used as an alternative to a continuous horizontal drain, as shown in figure 8-11, when the drain material is costly. The cross-sectional area of the stringer drains must be sufficient to satisfy the discharge requirements. The stringer drain may be constructed either by trenching into the embankment or foundation for narrow (6 ft) widths or by placing the adjacent impervious fill and then the drain material for wider (50 ft) widths (U. S. Army Engineer District, Kansas City 1974, 1978). In either case, the side slopes of the stringer drains should be sloped instead of
vertical (see figure 8-11) to avoid stress concentrations which could cause vertical transverse cracks. The stringer drain material must be thoroughly compacted (see paragraph 8-4c) to ensure that consolidation does not occur upon saturation leaving an open seepage conduit in the top of the trench, bridged by the overlying embankment, and susceptible to progressive erosion (Jansen 1980). The downstream end of horizontal drains and stringer drains must be able to discharge freely and must be protected against siltation and erosion. This may be accomplished by providing a weighted filter (riprap overlying bedding material) or a toe drain as shown in figure 8-12 (U. S. Army Engineer District, Mobile 1965 and U. S. Army Engineer District, Tulsa 1975). The toe drain has the advantage of lower maintenance requirements and preventing the development of localized wet areas at the surface along the downstream toe of the embankment.

8-6. **Seepage Control Against Earthquake Effects.** For dams located where earthquake effects are likely, there are several considerations which can lead to increased seepage control and safety. The core material should have a high resistance to erosion (Arulanandan and Perry 1983). Relatively wide transition and filter zones adjacent to the core and extending for the full height of the dam can be used. Additional screening and compaction of outer zones or shells will increase permeability and shear strength, respectively. Geometric considerations include using a vertical instead of inclined core, wider dam crest, increased freeboard, and flatter embankment slopes, and flaring the embankment at the abutments (Sherard 1966, 1967).
a. Weighted filter (courtesy of U. S. Army Engineer District, Mobile)

b. Toe drain (courtesy of U. S. Army Engineer District, Tulsa)

Figure 8-12. Protection of downstream end of horizontal and stringer drains
CHAPTER 9
SEEPAGE CONTROL IN EARTH FOUNDATIONS

9-1. **General.** All dams on earth foundations are subject to underseepage. Seepage control in earth foundations is necessary to prevent excessive uplift pressures and piping through the foundation (seepage control in earth abutments is given in Chapter 10). The purpose of the project, i.e., long-term storage, flood control, hydropower, etc., may impose limitations on the allowable quantity of seepage. Generally, siltation of the reservoir with time will tend to diminish underseepage (U. S. Army Engineer Division, Ohio River 1945). Conversely, the use of some underseepage control methods such as relief wells and toe drains may increase the quantity of underseepage (Sowers 1962).

9-2. **Selection of Method for Seepage Control.** The methods for control of underseepage in dam foundations are horizontal drains, cutoffs (compacted backfill trenches, slurry walls, concrete walls, and steel sheetpiling), upstream impervious blankets, downstream seepage berms, toe drains, and relief wells. To select an underseepage control method for a particular dam and foundation, the relative merits and efficiency of different methods should be evaluated by means of flow nets or approximate methods as described in Chapter 4 and Appendix B, respectively. As shown in table 9-1, the changes in the quantity of underseepage, factor of safety against uplift, and uplift pressures at various locations should be determined for each particular dam and foundation. Since the anisotropy ratio of the foundation has a significant influence on the results of the underseepage analysis, this parameter should be varied as appropriate \( \left( \frac{K_H}{K_V} = 1, 10, 25, 100 \right) \) to cover the possible range of expected field conditions.

9-3. **Horizontal Drains.** As mentioned previously in Chapter 8, horizontal drains are used to control seepage through the embankment and to prevent excessive uplift pressures in the foundation. As shown in figure 9-1, the use of the horizontal drain significantly reduces the uplift pressure in the foundation under the downstream portion of the dam. The computation of uplift pressure was illustrated previously in figure 4-15. Figure 9-1 also shows that the horizontal drain increases the quantity of seepage under the dam.

9-4. **Cutoffs.**

   a. **Complete Versus Partial Cutoff.** When the dam foundation consists of a relatively thick deposit of pervious alluvium, the designer must decide whether to make a complete cutoff (compacted backfill trench, slurry trench, or concrete wall) or allow a certain amount of underseepage to occur under controlled conditions (partial cutoff, upstream impervious blanket, downstream seepage berm, toe trench drain, or relief walls). In some cases, where the alluvium is not very deep or the water is very valuable, it may be obvious

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(1) Steel sheetpiling is not recommended to prevent underseepage but is used to confine the foundation soil and prevent piping.
Table 9-1. Selection of Underseepage Control Method

<table>
<thead>
<tr>
<th>Method</th>
<th>Changes in Quantity of Underseepage</th>
<th>$F_s = \frac{c}{i}$ (a)</th>
<th>Uplift Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal Drain</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Complete Cutoff</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Partial Cutoff</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upstream Impervious Blanket</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Downstream Seepage Berm</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Toe Drain</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relief Wells</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Factor of safety against uplift at downstream toe of dam given by the ratio of the critical hydraulic gradient to the vertical component of the exit gradient.

(Prepared by WES)
after relatively little study that a complete cutoff is justified. For example, the hydropower requirements of the Clarence Cannon Dam, Missouri, indicated that a complete cutoff was required to sustain power requirements during periods of little or no rainfall (U. S. Army Engineer District, St. Louis 1969). In many cases, where the cost of a complete cutoff is great and where the amount of underseepage without a complete cutoff is problemati-
cal, the decision is not easy. Factors which govern the decision for the type of underseepage control measure to be used are (Sherard 1968):

1. Economic comparison of the value of the water or hydropower which may be lost versus the cost of the complete cutoff.
(2) The resistance of the foundation alluvium with respect to potential progressive backward erosion of leaks or piping. If the foundation contains layers of fine sand or cohesionless silt, and particularly if these soils are exposed on the surface of the valley floor or walls, a complete cutoff is more desirable than if the foundation is basically gravelly (or even coarse sand). If a large leak develops in a relatively coarse alluvium, in all probability it will be safe against progressive backward erosion, but even a small concentrated leak emerging below the dam in fine cohesionless soils can be hazardous.

(3) If the tailwater conditions are such that ponds of water exist downstream of the dam so that underseepage would emerge underwater and could not be observed, it is desirable to be more conservative in evaluating the need for a complete seepage cutoff.

(4) The amount of silt and clay sized particles in suspension in the river water which contributes to siltation of the reservoir with time and tends to diminish underseepage.

Theory and model tests indicate that it is necessary for a cutoff to penetrate a homogeneous isotropic foundation at least 95 percent of the full depth before there is any appreciable reduction in seepage beneath an earth embankment as shown in figure 9-2 (Telling, Menzies, and Coulthord 1978; and Mansur and Perret 1949). The effectiveness of the partial cutoff in reducing the quantity of underseepage decreases as the ratio of the width of the dam to the depth of penetration of the cutoff increases (see figure 9-2). Partial cutoffs are effective only when they extend down into an intermediate stratum of lower permeability. This stratum must be continuous across the valley foundation to ensure that three-dimensional seepage around a discontinuous stratum does not negate the effectiveness of the partial cutoff.

b. Efficiency of Cutoffs. The effectiveness of the cutoff is assessed either in terms of the flow efficiency (Casagrande 1961)

\[ E_q = \frac{Q_o - Q}{Q_o} \]  

(9-1)

where

\[ E_q = \text{flow efficiency of cutoff} \]

\[ Q_o = \text{rate of underseepage without cutoff} \]

\[ Q = \text{rate of underseepage with cutoff} \]

or head efficiency (Lane and Wohlt 1961)

\[ E_H = \frac{h}{H} \]  

(9-2)
EM 1110-2-1901
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where

\[ E_h = \text{head efficiency of cutoff} \]
\[ h = \text{head loss between points immediately upstream and downstream of the cutoff wall at its junction with the base of the dam} \]
\[ H = \text{head loss across the dam} \]

The head efficiency is more widely used because the field performance may be established from piezometric data taken during construction, before and during initial filling of the reservoir, and subsequently as frequently as necessary to determine changes that are occurring and to assess their implications with respect to safety of the dam, as described in Chapter 13. The flow efficiency may only be approximated since the rate of underseepage without the cutoff cannot be directly established and since it is difficult to measure the rate of underseepage with the cutoff because, except for special cases, only part of the underseepage discharges at the ground surface immediately downstream of the dam (Telling, Menzies, and Simons 1978a and Marsal and Resendiz 1971). The flow efficiency of a compacted backfill partial cutoff in a foundation of permeable soils of moderate thickness overlying an impervious rock is shown in figure 9-3. This figure also illustrates the high seepage gradients that occur along the base of the cutoff and on its downstream face in both the foundation and embankment zones. Suitable filters must be provided to prevent piping of soil at faces A-B-C in figure 9-3a and 9-3b (Cedergren 1977 and Klohn 1979).

As shown in figure 9-4, a partial cutoff in a homogeneous isotropic foundation will lower the line of seepage in the downstream embankment somewhat but exit gradients at the downstream toe (as reflected by the distance between the equipotential lines) are reduced only slightly (Cedergren 1973). When the pervious foundation is cut off by a compacted backfill or slurry trench, \(^{(1)}\) the rate of underseepage may be estimated by (Ambraseys 1963 and Marsal and Resendiz 1971)

\[
\frac{Q_o}{K_o H} = \frac{1}{0.88 + \frac{B}{D} + \frac{(K_o/K - 1)}{E/D}} \tag{9-3}
\]

where

\[ Q_o = \text{rate of underseepage in m}^3/\text{set per running meter of dam} \]
\[ K_o = \text{permeability of the foundation in m/sec} \]

\(^{(1)}\) This approach neglects the contribution of the filter cake that forms on the trench walls to the overall slurry trench permeability. When the permeability of the backfill placed in the trench is high, the overall slurry trench permeability will be controlled by the filter cake (D'Appolonia 1980).
H = head of water in the reservoir in m
B = width of the base of the dam in m
D = thickness of the foundation in m
k = permeability of the compacted backfill or slurry trench backfill in m/sec
E = thickness of the cutoff in m

For a concrete wall or steel sheetpiling with defects (openings in the cutoff) the rate of underseepage per unit length of cutoff is given by (Ambraseys 1963 and Marsal and Resendiz 1971)

\[
\frac{Q_0}{K_0 H} = \frac{1}{0.88 + \frac{B}{D} + \left(\frac{D}{W} - 1\right) \frac{E}{D}}
\]

(9-4)

where

W = total area of openings in m²

Figure 9-5 compares the rate of underseepage for an impervious upstream blanket, compacted backfill trench or slurry trench, and concrete wall or steel sheetpiling with defects. As shown in figure 9-5, the rate of underseepage loss is the same for an impervious upstream blanket as for a compacted backfill trench or slurry trench provided \(B' = \left(\frac{K_0}{k} - 1\right) E\).

If \(K\) is relatively high, assuming \(\frac{K_0}{k} = 50\) gives \(B' = 49 E\). Such computations allow preliminary cost estimates to be made to determine whether an impervious upstream blanket is preferable over a compacted backfill trench or slurry trench. Figure 9-6 can be used to determine the relative magnitudes of the length of the impervious upstream blanket, the thickness of the compacted backfill trench or slurry trench, or the area of the defects (openings) in the concrete wall or steel sheetpiling that would result in the same rate of underseepage for a given dam.

c. Compacted Backfill Trench. The most positive method for control of underseepage consists of excavating a trench beneath the impervious zone of the embankment through pervious foundation strata and backfilling it with compacted impervious material. The compacted backfill trench is the only method for control of underseepage which provides a full-scale exploration trench that allows the designer to see the actual natural conditions and to adjust the design accordingly, permits treatment of exposed bedrock as necessary, provides access for installation of filters to control seepage and prevent piping of soil at interfaces, and allows high quality backfilling operations to be carried out (U. S. Army Engineer District, St. Louis 1969 and Cedergren 1977).
Figure 9-2. Effect of depth of penetration of partial cutoff on seepage reduction for a homogeneous isotropic foundation (prepared by WES)
a. Partial cutoff

b. Complete cutoff

c. Relationship between quantity of seepage and depth of penetration of partial cutoff

Figure 9-3. Efficiency of a compacted backfill trench partial cutoff in reducing the quantity of underseepage (courtesy of John Wiley and Sons)
Figure 9-4. Effect of depth of penetration of partial cutoff on the height of the line of seepage in the downstream embankment and exit gradient at the toe for a homogeneous isotropic foundation (courtesy of John Wiley and Sons).
Material and compaction requirements are the same as those for the impervious section of the dam (EM 1110-2-1911). When constructing a complete cutoff (see para 9-4a), the trench must fully penetrate the pervious foundation and be carried a short distance into unweathered and relatively impermeable foundation soil or rock. To ensure an adequate seepage cutoff, the base width should be at least one-fourth the maximum difference between the reservoir and tailwater elevations but not less than 20 ft, and should be wider if the foundation material under the cutoff is considered marginal in respect to imperviousness. As previously mentioned (see para 9-4b), high seepage gradients occur along the base of the cutoff and on its downstream face in both the foundation and embankment zones. Suitable filters must be provided (see Appendix D for design of filters) to prevent piping of soil at these interfaces. The trench
excavation must be kept dry to permit proper placement and compaction of the impervious backfill. Dewatering systems of wellpoints or deep wells are generally required during excavation and backfill operations when below groundwater levels (TM 5-818-5). Because construction of an open cutoff trench with dewatering is a costly procedure, the trend has been toward use of the slurry trench method of construction (EM 1110-2-2300 and Cedergren 1977).

d. Slurry Trench.

(1) Introduction. When the cost of dewatering and/or the depth of the pervious foundation render the compacted backfill trench too costly and/or impractical, the slurry trench cutoff may be a viable method for control of
underseepage. Using this method, a trench is excavated through the pervious foundation using a sodium bentonite clay (or Attapulgite clay in saline water) and water slurry to support the sides. The slurry-filled trench is backfilled by displacing the slurry with a backfill material that contains enough fines (material passing the No. 200 sieve) to make the cutoff relatively impervious but sufficient coarse particles to minimize settlement of the trench forming a soil-bentonite cutoff (sometimes called American method). Alternatively, cement may be introduced into the slurry-filled trench which is left to set or harden forming a cement-bentonite cutoff (sometimes called a grouted diaphragm wall or Coulis wall or European method). The slurry trench cutoff is not recommended when boulders, talus blocks on buried slopes, or open jointed rock exist in the foundation due to difficulties in excavating through the rock and slurry loss through the open joints. Where a slurry trench is relied upon for seepage control, the initial filling of the reservoir must be controlled and piezometers located both upstream and downstream of the cutoff must be read to determine if the slurry trench is performing as planned. If the cutoff is ineffective, remedial seepage control measures (see Chapter 12) must be installed prior to further raising of the reservoir pool (KM 1110-2-2300).

(a) History of Use. The first use of the slurry trench method of construction was by the U. S. Army Engineer District, Memphis, in September 1945, to form a partial cutoff along the Mississippi River levee on the Arkansas side of the river just below Memphis, Tennessee (Clay 1976 and Kramer 1946). The idea for the project probably evolved from the use at that time of puddle clay trench cutoffs combined with the use of drilling mud for advancing borings. A paddle wheel mixing device was constructed for making slurry from native clays. Trenches were dug to a 20-ft depth using a trenching machine and to a 35-ft depth using a dragline with a 100-ft boom and 2-cu-yd bucket. Backfill was mixed in windrows at the site from hauled-in clay gravel and native materials and pushed into the trench by a bulldozer when the length of the trench was equal to about twice the trench depth. BG Hans Kramer foresaw the use of the slurry trench method for the construction of cutoffs for earth dams. It is amazing that after 38 years, the technique is still about the same as it was when first developed by the Memphis District. A soil-bentonite cutoff was constructed under the Kennewick Levee adjacent to the Columbia River as part of the McNary Dam Project in Washington by the Walla Walla District in 1952 (Jones 1961). The first application of a soil-bentonite slurry trench cutoff for control of underseepage at a major earth dam was at Wanapum Dam on the Columbia River in Washington in 1959 (La Russo 1963). Subsequently, soil-bentonite cutoffs have been used for control of underseepage at a number of dams as shown in table 9-2. The cement-bentonite slurry trench cutoff was first used to tie into the abutment zones at the Razaza Dam on the Euphrates River in Iraq in 1969 (Soletanche 1969). Subsequently, cement-bentonite cutoffs were installed as remedial seepage control measures through the embankment and foundation of four existing dams in Mexico from 1970 to 1972 (Soletanche 1970, 1971, 1971-1972, 1972). As shown in table 2, the first cement-bentonite cutoff in the United States was constructed at the Tilden Tailings Project to store tailings from the Tilden Mine in Michigan in 1976 (Meier and Rettberg 1978). The first cement-bentonite cutoff constructed at a dam on a river retaining a reservoir in the United States was completed in 1978 at the Elgo Dam (formerly the San Carlos Dam) in Arizona (Anonymous 1978 and Miller and Salzman 1980).
**Table 9-2. Comparison of Slurry Trench Cutoffs**

<table>
<thead>
<tr>
<th>Project</th>
<th>Location</th>
<th>Owner</th>
<th>Date Constructed</th>
<th>Foundation Material</th>
<th>Max. Differential Head ft</th>
<th>Cutoff Width ft</th>
<th>Depth ft</th>
<th>Location</th>
<th>Max. Head Cutoff Width</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kennewick Levee, McHenry Dam</td>
<td>Columbia River, Wash.</td>
<td>Corps of Engineers</td>
<td>1952</td>
<td>Sandy or silty gravel with zones of open gravel</td>
<td>15</td>
<td>6</td>
<td>22</td>
<td>Center of dam</td>
<td>2.5</td>
<td>Jones 1964</td>
</tr>
<tr>
<td>Manapum Dam</td>
<td>Columbia River, Wash.</td>
<td>Public Utility District 1, Grant County, Wash.</td>
<td>1962</td>
<td>Sandy gravel and gravelly sands underlain by open work gravels</td>
<td>88</td>
<td>10</td>
<td>80</td>
<td>Center of dam</td>
<td>8.8</td>
<td>La Russo 1963</td>
</tr>
<tr>
<td>Wells Dam</td>
<td>Columbia River, Wash.</td>
<td>Public Utility District 1, Douglas County, Wash.</td>
<td>1964</td>
<td>Ferrous gravel</td>
<td>70</td>
<td>8</td>
<td>80</td>
<td>Center of dam</td>
<td>8.8</td>
<td>Jones 1967</td>
</tr>
<tr>
<td>Yards Creek - Lower Reservoir</td>
<td>New Jersey</td>
<td>Public Service</td>
<td>1966</td>
<td>Sands, gravels, cobbles, and boulders</td>
<td>55</td>
<td>8</td>
<td>40</td>
<td>Center of dam</td>
<td>6.9</td>
<td>Jones 1967</td>
</tr>
<tr>
<td>Comanche Dam - Bike 2</td>
<td>Hokeleeum River, Calif.</td>
<td>East Bay Municipal District</td>
<td>1966</td>
<td>Upper stratum of clayey silts, silts, and clayey sands. Lower stratum of sand over zone of gravel</td>
<td>45</td>
<td>8</td>
<td>95</td>
<td>Upstream toe of dam</td>
<td>5.6</td>
<td>Anton and Dayton 1972</td>
</tr>
<tr>
<td>Mill Site Dam</td>
<td>Ferron Creek, Utah</td>
<td>Soil Conservation Service</td>
<td>1969</td>
<td>Sandy gravel</td>
<td>73</td>
<td>8</td>
<td>68</td>
<td>Upstream toe of dam</td>
<td>9.1</td>
<td>Hanson 1972</td>
</tr>
<tr>
<td>Plum Creek Dam</td>
<td>Plum Creek, Wisc.</td>
<td>Soil Conservation Service</td>
<td>1973</td>
<td>Sands, gravelly sands, silty gravel, silty sands</td>
<td>34</td>
<td>5</td>
<td>40</td>
<td>Center of dam</td>
<td>10.8</td>
<td>Knobach and Dingel 1974</td>
</tr>
<tr>
<td>Upper Big Blue Dam</td>
<td>Upper Big Blue River, Ind.</td>
<td>Soil Conservation Service</td>
<td>1978</td>
<td>Sands and gravels</td>
<td>62</td>
<td>4</td>
<td>40</td>
<td>Center of dam</td>
<td>15.5</td>
<td>Bloom, Dynes, Closesett 1979</td>
</tr>
<tr>
<td>Addicks Dam(a)</td>
<td>Buffalo Bayou, Tex.</td>
<td>Corps of Engineers</td>
<td>1979</td>
<td>Silty sands</td>
<td>30</td>
<td>3</td>
<td>70</td>
<td>Varies</td>
<td>10.0</td>
<td>U.S. Army Engr. Dist, Galveston 1983 (same as Barker Dam)</td>
</tr>
<tr>
<td>Barker Dam(a)</td>
<td>Buffalo Bayou, Tex.</td>
<td>Corps of Engineers</td>
<td>1979</td>
<td>Sandy clay, clayey sand, silty sand</td>
<td>26</td>
<td>3</td>
<td>55</td>
<td>Center of dam</td>
<td>8.0</td>
<td>Galveston 1983 (same as Barker Dam)</td>
</tr>
</tbody>
</table>

**Cement-bentonite Slurry Trench**

| Tilden Tailings               | Tilden Mine, Mich.        | Cleveland-Cliffs Iron Company | 1976             | Sand interspersed with gravel | 100(b)            | 2                | 80       | Upstream toe of dam    | 50.0      | Heier and Berzig 1978    |
| Elgo Dam (formerly San Carlos Dam) | San Carlos River, Ariz. | San Carlos Apache Tribe      | 1978             | Sands and gravels          | 66             | 3                | 68       | Center of dam          | 21.0      | Hillier and Salzman 1980  |

(a) Slurry trench cutoff installed as remedial seepage control for existing dam (see Chapter 12).
(b) Planned maximum differential head when tailings dam is at final height (45 ft in February 1983).
EM 1110-2-1901
30 Sep 86


(c) Comparison of Soil-Bentonite and Cement-Bentonite Slurry Trench Cutoffs. A comparison of soil-bentonite and cement-bentonite slurry trench cutoffs is given in table 9-3. The soil-bentonite slurry trench cutoff is generally the most economical if the cost of backfill is not prohibitive. For deep cutoffs where the foundation is prone to failure during excavation, the cement-bentonite slurry trench cutoff is more applicable.

(d) Location of Cutoff. Normally, the slurry trench should be located under or near the upstream toe of the dam (EM 1110-2-2300). An upstream location provides access for future treatment provided the reservoir could be drawn down and facilitates stage construction by permitting placement of a downstream shell followed by an upstream core tied into the slurry trench. For stability analysis, a soil-bentonite slurry trench cutoff should be considered to have zero shear strength and exert only a hydrostatic force to resist failure of the embankment (U. S. Army Engineer District, Savannah 1968). If the slurry trench is located under a central core, consolidation of the slurry trench backfill combined with arching of the core material immediately above the slurry trench may result in the opening of a cavity under the dam with possible leakage along the contact. If a central location for the slurry trench is dictated by other factors, some possible benefits are obtained by flaring the top of the trench to provide a transition between the cutoff and the core. Also, the slurry trench can be constructed and allowed to settle before placement of the embankment (Jones 1967 and Jansen 1968). When the groundwater table is located some distance beneath the ground surface, it is usually more economical to excavate a conventional open trench with stable side slopes with the trench bottom a few feet above the highest level of ground water expected during the construction period, as was done at West Point Dam, Alabama and Georgia (U. S. Army Engineer District, Savannah 1968). The bottom of the open trench provides a working level from which the slurry trench may be constructed. Also, this prevents the problem of significant amounts of slurry being lost into the excavated trench above the ground-water table. If the ground-water table is located near the ground surface, compacted impervious fill should be placed in order to raise the level of the slurry trench to maintain the level of slurry in the trench a sufficient distance above the ground-water level (Jones 1967).

(e) Stability of the Trench. In cohesionless soils the penetration of the slurry into the wall of the slurry trench excavation forms a relatively impervious filter cake on which the hydrostatic pressure of the slurry can act. The depth of penetration ranges from 1 to 3 in. in sand, 3 to 6 in. in sand and gravel, and up to 12 in. in gravel, depending on the gradation. The main stabilizing force supporting the slurry trench excavation is the hydrostatic pressure exerted on the trench walls. For a slurry trench excavated in a homogeneous clay, remaining open only for a few days to permit placement of
Table 9-3. Comparison of Soil-Bentonite and Cement-Bentonite Slurry Trench Cutoffs(a)

<table>
<thead>
<tr>
<th>Item</th>
<th>Soil Bentonite Slurry Trench Cutoff</th>
<th>Cement-Bentonite Slurry Trench Cutoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Excavation</td>
<td>Long slope of backfill requires trenching continuously in one direction. Can accommodate interruptions in construction.</td>
<td>Construction sequence is more flexible to meet site constraints. Excavation for each panel should progress uninterrupted so each panel is completed before the slurry begins to set.</td>
</tr>
<tr>
<td></td>
<td>Difficult to work with coarse gravels, with backhoe and/or dragline, which settle out and accumulate on the trench bottom reducing the certainty of a tight seal.</td>
<td>Easier to remove coarse gravels with clam shell mounted on a rigid Kelly bar, from trench bottom. Better suited for excavation in areas prone to failure.</td>
</tr>
<tr>
<td></td>
<td>Wider trench required to prevent segregation during backfilling (&gt; 3 ft) and prevent piping failure of the slurry trench backfill into the adjacent soil downstream of the cutoff (1 ft width for each 10 ft of differential head)</td>
<td>Narrower trench (2 – 3 ft) may be used.</td>
</tr>
<tr>
<td>2. Slurry</td>
<td>Mixture of bentonite and water used to support excavation during trenching. Slurry displaced by backfill is desanded and reused. Upon completion of trench, unused slurry must be disposed of in an environmentally acceptable fashion.</td>
<td>Mixture of cement, bentonite, and water used to support excavation for panel and later sets to form permanent backfill. No disposal or desanding of slurry required.</td>
</tr>
<tr>
<td>3. Backfill</td>
<td>Mixture of slurry plus excavated material from trench and/or imported select backfill materials</td>
<td>None (see 2). Therefore not dependent on availability or quality of soil for backfill and more suitable for work in confined areas.</td>
</tr>
<tr>
<td>4. Properties of completed cutoff:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Permeability</td>
<td>$10^{-7}$ cm/sec ($\leq$ 1 percent bentonite)</td>
<td>$10^{-6}$ cm/sec</td>
</tr>
<tr>
<td>b. Strength</td>
<td>Little field data available - assumed zero for design purposes.</td>
<td>15 – 20 lbs/sq in. unconfined compressive strength. Will support construction loadings or future structures after several weeks.</td>
</tr>
<tr>
<td>c. Consolidation</td>
<td>Some consolidation with time.</td>
<td>No significant consolidation with time.</td>
</tr>
<tr>
<td>5. Susceptible to attack by sulphates or other chemicals in groundwater</td>
<td>No</td>
<td>Yes – cement may be.</td>
</tr>
<tr>
<td>6. Relative cost</td>
<td>Generally lower if cost of backfill is not prohibitive (see 3)</td>
<td>Generally higher due to cost of cement. More competitive as depth of trench increases due to narrower trench width required.</td>
</tr>
</tbody>
</table>

(a) Courtesy of Engineering Construction International, Inc. 132, 249, American Society of Civil Engineers 222.
backfill material, the factor of safety against instability is (Nash and Jones 1963)

\[ F = \frac{4 c_u}{H(\gamma - \gamma_s)} \]  

(9-5)

where

\( F \) = factor of safety

\( c_u \) = undrained cohesion

\( H \) = depth of the trench

\( \gamma \) = unit weight of the soil

\( \gamma_s \) = unit weight of the slurry

For a slurry trench excavated in dry cohesionless soil (Nash and Jones 1963)

\[ F = \frac{2(\gamma \times \gamma_s)^{0.5} \tan \phi}{\gamma - \gamma_s} \]  

(9-6)

where \( \phi \) = angle of internal friction.

For a slurry trench excavated in a saturated cohesionless soil with the ground-water table and slurry level in the excavation both at the ground surface (Nash and Jones 1963)

\[ F = \frac{2(\gamma' \times \gamma'_s)^{0.5} \tan \phi}{\gamma' - \gamma'_s} \]  

(9-7)

where

\( \gamma' \) = effective unit weight of the soil

\( \gamma'_s \) = effective unit weight of the slurry

\( \phi \) = effective angle of internal friction

For arbitrary levels of ground water and slurry in cohesionless soil, as shown in figure 9-7a, a slightly conservative (neglects arching effect of short
trenches and stabilization of the soil adjacent to the trench face due to slurry penetration and gelation) estimate of the slurry density required to ensure stability of the trench is (Morgenstern and Amir-Tahmasseb 1965)

\[
n^2 \gamma_s \frac{\gamma}{\gamma_w} \frac{\cot \alpha (\sin \alpha - \cos \alpha \tan \phi') + m^2 \csc \alpha \tan \phi'}{\cos \alpha + \sin \alpha \tan \phi'} = (9-8)
\]

where

- \( n \) = defined in figure 9-7a
- \( \gamma_s \) = unit weight of the slurry
- \( \gamma_w \) = unit weight of water
- \( \gamma \) = unit weight of the soil
- \( \alpha \) = angle of inclination of the wedge of soil at the point of slipping, in practice assumed to be equal to 45° + \( \phi'/2 \)
- \( \phi' \) = effective angle of internal friction
- \( m \) = defined in figure 9-7a

Equation 9-6 may be solved by use of the nomograph shown in figure 9-7b (Duguid et al. 1971).

(2) Slurry. The slurry has three basic functions in slurry trench construction (Ryan 1977):

- To hold the trench open and maintain a stable excavation.
- To be fluid enough to permit passage of the excavating equipment and to allow placement of the backfill (for the cement-bentonite slurry trench, there is no backfill).
- To form a filter cake to enhance the low permeability of the completed trench.

(a) Materials. As a general rule sodium montmorillonite in powder form (Wyoming-type bentonite) is used for slurry trench construction. However, when salt water is present Attapulgite clay is used to avoid flocculation (Spooner et al. 1982). Specifications for both bentonite and Attapulgite are given by the American Petroleum Institute (American Petroleum Institute 1981). Each shipment of bentonite or Attapulgite should be checked for compliance with the specifications. At Saylorville Dam, Iowa, changes in slurry properties were traced to lower quality bentonite which was mined from different beds (U. S. Army Engineer District, Rock Island 19788). No chemically treated
a. Slurry trench typical section

b. Nomograph for slurry trench stability

Figure 9-7. Determination of slurry trench stability in cohesionless soil (courtesy of National Research Council of Canada).
bentonite should be used for slurry trench construction. The pH of the water used for mixing with the bentonite should equal \(7.0 \pm 1.0\). Water hardness should not exceed 50 ppm (parts per million). Total dissolved solids should not exceed 500 ppm. The amount of oil, organics, or other deleterious substances should be limited to no more than 50 ppm each (Stanley Consultants, Inc., and Woodward-Clyde Consultants 1977). If the use of poor quality water cannot be avoided, it will require more bentonite and longer mixing times to achieve the desired properties.

(b) Mixing. The methods used to prepare the bentonite slurry vary with project size and layout. Always add the bentonite to the water, never the water to the bentonite. For small jobs the batch system is used where specific quantities of water and bentonite are placed in a tank and mixed at high speeds with a circulation pump or paddle mixer and the slurry is discharged into the trench. Mixing is usually complete in a matter of minutes for the 2- to 5-cu-yd batch produced by this method. The most commonly used method is the flash or Venturi mixer and circulation ponds which is well adapted for bulk handling of large slurry volumes. A flash mixer introduces dry bentonite into a turbulent water jet which discharges into a low speed circulation pond. When Marsh Funnel viscosity readings stabilize, the slurry is stored in a second pond prior to using the trench (Spooner et al. 1982 and D'Appolonia 1980).

(c) Properties. Tests of bentonite quality must be conducted for each rail car or truck load delivered. The minimum acceptable viscosity of a slurry made with the bentonite is 40 seconds Marsh funnel viscosity at 65° F. The fresh slurry shall have a minimum Marsh funnel viscosity of 40 seconds at 65° F and a pH of from 7 to 10, a bentonite content of from 3 to 7 percent by dry weight (depending on the grade of bentonite), a unit weight of from 1.0 to 1.04 g/cm\(^3\) (about 65 lb/ft\(^3\)), and the filtrate or water loss shall not be greater than 20 at 100 lb/in.\(^2\) x cm\(^3\) in 30 minutes (Spooner et al. 1982 and U. S. Army Engineer District, Savannah 1968). The slurry in the trench should be sampled at least twice daily with samples taken from the top of the trench and at 10-ft vertical, 50-ft horizontal intervals along the trench center line. During all stages of construction the minimum acceptable viscosity of the slurry shall be 40 seconds Marsh funnel viscosity at 65° F. The minimum in-trench slurry unit weight is based upon trench stability considerations (see Equation 9-8 and Figure 9-7). The maximum in-trench slurry density is 85/ft\(^3\) to avoid buildup of sediment beyond the slurry capacity to hold it in suspension in the trench during excavation (Clough 1978).

(d) Quality Control Testing. In order to mix and maintain a proper slurry to hold the trench open during excavation and form a filter cake for the soil-bentonite slurry trench, quality control testing must be performed. The property, frequency, standard (if any), and specified value for slurries and their components are given in table 9-4. The quality of the mixing water used can influence the slurry trench characteristics. If the specific values for mixing water quality are not met, the bentonite will flocculate and settle out and not form the filter cake on the sides of the trench. Poor quality mixing water will increase the set time for cement-bentonite slurry trench cutoffs. The bentonite is tested to be sure it will have the minimum viscosity required to keep the soil in suspension. The slurry is tested both after
Table 9-4. Summary of Quality Control Tests for Slurry Trench Cutoff

<table>
<thead>
<tr>
<th>Category or Material</th>
<th>Property</th>
<th>Frequency</th>
<th>Standard</th>
<th>Specified Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixing Water</td>
<td>pH</td>
<td>Once per month</td>
<td>API RP 13B</td>
<td>7.0 ± 1.0</td>
</tr>
<tr>
<td></td>
<td>Hardness</td>
<td>Once per month</td>
<td>API RP 13B</td>
<td>≤ 50 ppm</td>
</tr>
<tr>
<td></td>
<td>Total dissolved solids</td>
<td>Once per month</td>
<td>API RP 13B</td>
<td>≤ 500 ppm</td>
</tr>
<tr>
<td></td>
<td>Oil, organics, etc.</td>
<td>Once per month</td>
<td>API RP 13B</td>
<td>≤ 50 ppm each</td>
</tr>
<tr>
<td>Bentonite</td>
<td>Viscosity</td>
<td>Each railroad car or truck load</td>
<td>API RP 13A</td>
<td>≥ 40 seconds at 65°F</td>
</tr>
<tr>
<td>Bentonite slurry (after mixing)</td>
<td>Viscosity</td>
<td>Twice per day</td>
<td>API RP 13B</td>
<td>≥ 40 seconds at 65°F</td>
</tr>
<tr>
<td></td>
<td>pH</td>
<td>Twice per day</td>
<td>API RP 13A</td>
<td>7 - 10</td>
</tr>
<tr>
<td></td>
<td>Bentonite content</td>
<td>Twice per day</td>
<td>API RP 13B</td>
<td>1.01 - 1.04 g/cm³</td>
</tr>
<tr>
<td></td>
<td>Unit weight</td>
<td>Twice per day</td>
<td>API RP 13B</td>
<td>≥ 20 cm³ at 100 psi in 30 min</td>
</tr>
<tr>
<td></td>
<td>Filtrate or water loss</td>
<td>Twice per day</td>
<td>API RP 13B</td>
<td>&gt; 40 seconds at 65°F</td>
</tr>
<tr>
<td>Bentonite slurry (in trench)</td>
<td>Viscosity</td>
<td>Twice per day</td>
<td>API RP 13B</td>
<td>Minimum (c) - 85 pcf</td>
</tr>
<tr>
<td></td>
<td>Unit weight</td>
<td>Twice per day</td>
<td>API RP 13B</td>
<td></td>
</tr>
<tr>
<td>Cement</td>
<td>Slump</td>
<td>Twice per day</td>
<td>ASTM C-143 (d)</td>
<td>2 - 5 in.</td>
</tr>
<tr>
<td>Cement-bentonite slurry (after mixing)</td>
<td>Viscosity</td>
<td>Twice per day</td>
<td>API RP 13B</td>
<td>40-50 seconds at 65°F</td>
</tr>
<tr>
<td></td>
<td>pH</td>
<td>Twice per day</td>
<td>API RP 13A</td>
<td>10 - 13</td>
</tr>
<tr>
<td></td>
<td>Bentonite content</td>
<td>Twice per day</td>
<td>None (b)</td>
<td>4 - 7 percent</td>
</tr>
<tr>
<td></td>
<td>Unit weight</td>
<td>Twice per day</td>
<td>API RP 13B</td>
<td>1.03 - 1.04 g/cm³</td>
</tr>
<tr>
<td></td>
<td>Filtrate or water loss</td>
<td>Twice per day</td>
<td>API RP 13B</td>
<td>≤ 165 cm³ at 100 psi in 30 min</td>
</tr>
<tr>
<td></td>
<td>Cement/water ratio</td>
<td>Twice per day</td>
<td>None (b)</td>
<td>0.17 - 0.20</td>
</tr>
</tbody>
</table>

(a) American Petroleum Institute specification.
(b) Can be checked by evaporating the water from a known volume of slurry and weighing the remaining solids.
(c) Minimum in trench slurry unit weight is based upon trench stability considerations.
(d) American Society for Testing and Materials Standard.
mixing and in the trench to determine that it is dense enough to stabilize the trench, but not so dense as to cause the backfill to settle too loosely, and that it has sufficient viscosity to maintain cuttings in suspension (Stanley Consultants, Inc., and Woodward-Clyde Consultants 1976).

(3) Excavation, Mixing, and Backfilling.

(a) Excavation. The preferred method of trench excavation depends upon the required depth of the slurry trench cutoff, the nature of the subsurface materials, and access to the trench at the ground surface. It is important to ensure that the equipment used can maintain a continuous excavation line to the total depth required. At depths less than about 50 ft, backhoes are generally the most rapid and least costly excavation method. Modified backhoes with an extended dipper stick, modified engine, and counter-weighted frame can excavate to about 80 ft deep. Draglines with weighted (> 10,000 lb) buckets, which have been used in the depth range 60 to 80 ft, have been replaced by more efficient extended backhoes. The clamshell bucket can excavate to depths in excess of 150 ft. The clamshell may be mechanically operated attached to a crane or hydraulically operated attached to a Kelly bar. On larger jobs, the backhoe may be used to excavate the first 50 ft followed by a clamshell bucket to excavate the primary and secondary panels to the impervious zone. Regardless of the equipment used, it should be capable of excavating a trench of the desired width in a single pass in order to obtain a fairly consistent trench width. The bucket used should be nonperforated to allow retention and removal of sand particles from the trench. The continuity of the trench is tested by passing the bucket or clamshell of the excavating tool vertically and horizontally along each segment of the trench before it is backfilled. Whatever excavation method is used, it is important that good communications are maintained with the operator of the excavation equipment since abnormalities in the trench excavation are usually noticed first by the equipment operator (D’Appolonia 1980; Spooner et al. 1982; Bloom, Dynes, and Glossett 1979; Case International Company 1982; and Winter 1978). Soil-bentonite slurry trench cutoffs are excavated in a continuous trench as shown in figure 9-8a, while cement-bentonite slurry trench cutoffs are excavated in a continuous trench or in short sections or panels as shown in figure 9-8b. The cement-bentonite slurry begins to harden within 2 to 3 hours after mixing. Alternate panels are excavated under a cement-bentonite slurry and then allowed to partially set. Intervening panels are excavated also under a cement-bentonite slurry and a portion of the initial panel ends are removed to ensure continuity between adjacent panels. Construction delays can cause problems in setup of cement-bentonite slurry trench cutoffs because continued agitation of the cement-bentonite slurry (more than 24 hours) reduces the ability of the cement to set (Spooner et al. 1982).

(b) Bottom Treatment. The aquiclude used for the slurry wall foundations should be continuous, and relatively free of fractures and other pervious zones. The cutoff wall should extend a minimum of 2 ft into clay (or 1 ft into rock) to prevent weathered zones, desiccation cracks, or other geological features from permitting seepage under the cutoff (Spooner 1982). As the trench is excavated, heavier soil particles such as sand and gravel fall to the bottom of the trench. The amount of sand accumulation on the trench bottom depends upon the coarseness of the strata being excavated as well as the
Figure 9-8. Construction procedure for soil-bentonite and cement-bentonite cutoffs (courtesy of American Society of Civil Engineers) 

excavation technique used. Although this sand layer may not have a direct effect on trench stability, it may adversely affect the permeability of the slurry trench cutoff wall (Spooner 1982). An air lift pump should be used to remove the sand and gravel particles from the trench bottom prior to backfilling. When the slurry trench is keyed into a soil aquiclude after the trench bottom has been cleaned thoroughly, a minimum of one split-spoon sample shall be taken every 50 ft along the length of the trench to determine if additional excavation is required (Winter 1978).

(c) Backfill Mixing and Placement. A minimum of one day is required between trench excavation and backfilling in order to develop a low permeability filter cake on the trench walls (D'Appolonia 1980). Stockpiled material from the trench excavation and/or material from borrow areas are mixed and blended by windrowing, disc harrowing, bulldozing, or by blading to remove lumps of clay, sand, or gravel. The backfill is then sluiced with slurry (mixing with water shall not be permitted) and just prior to placement has a consistency of a wet concrete with a slump of 5 in. ± 1 in. tested in
accordance with ASTM C-143 \(^{(1)}\) (Winter 1978 and Ryan 1976). The backfill is placed continuously from the beginning of the trench in the direction of the excavation to the end of the trench. Free dropping of the backfill material through the slurry would produce segregation and is not allowed. Depending on the steepness of the excavated slope, it may be necessary to lower the initial backfill to the bottom of the trench with a crane and clamshell bucket until a slope at the angle of repose of the backfill has been formed from the bottom of the trench to the top of the excavation. The toe of the backfill slope is kept to within 50 to 150 ft of the leading edge of the excavation to minimize the open length of the slurry-supported trench while allowing enough space behind the excavation for cleaning the trench bottom. Additional backfill is placed by a bulldozer in such a manner that the backfill enters the trench and slides progressively down the slope of the previously placed backfill and produces a slope ranging from 1V on 5H to 1V on 10H. The slope of the backfill shall be measured with soundings starting at the toe of the backfill in the bottom of the trench and progressing up the backfill slope at 25-ft horizontal intervals. A set of soundings shall be made at least for every 25 ft horizontal advancement of backfill placement. Once the natural slope of the backfill is established during initial placement of the backfill, the slope should remain nearly the same. If the slope, or a portion of the slope, suddenly gets steeper, it could be an indication that sediment is being trapped or that the backfill has a pocket of relatively clean material (slurry not mixed in properly or was washed out). If the slope suddenly gets flatter, it could indicate that a pocket of slurry was trapped in the backfill or that the backfill does not contain sufficient sand or coarser material (Stanley Consultants, Inc., and Woodward-Clyde Consultants 1976; Winter 1978; and Ryan 1976).

(d) Temperature During Construction. The mixing and placing of backfill shall be limited to days when the air temperature is not less than 20°F. Even though the surface of the slurry trench freezes overnight, there will be no difficulty breaking through the surface ice and continuing excavation during the day. Frozen backfill or pieces of ice must never be placed in the trench (U. S. Army Engineer District, Rock Island 1978a and Jones 1967).

(e) Protection of Top of Trench. The top of the completed slurry trench cutoff should be immediately protected with a temporary 2- to 3-ft-thick blanket of moist impervious fill material to prevent drying of the backfill and formation of shrinkage cracks along which paths of seepage could easily develop. The layer is temporary and is removed once the embankment construction is started (Jones 1967; Stanley Consultants, Inc., and Woodward-Clyde Consultants 1977).

(4) Soil-Bentonite Slurry Trench Cutoff.

\(^{(1)}\) American Society for Testing and Materials standard. If a desirable backfilled slope (1V on 5H to 1V on 10H) cannot be maintained in the trench with a 5 in. ± 1 in. slump, the slump may be altered to meet construction conditions. Such was the case at the soil-bentonite slurry trench cutoff constructed at W. G. Huxtable Pumping Plant, Marianna, Arkansas (U. S. Army Engineer District, Memphis 1978).
(a) Design Considerations. The primary design parameters are blowout requirements, permeability, strength, and compressibility. The backfill material must not blow out into the surrounding pervious foundation under the maximum differential hydraulic head that will act on the slurry trench. The permeability is usually sufficiently low \((-10^{-7}\ \text{cm/sec for } > 1\ \text{percent bentonite})\) to reduce the seepage through the slurry trench cutoff to an acceptable value. Under most conditions, the only strength requirement for the slurry trench cutoff is to approximate the strength of the surrounding ground. The compressibility of the slurry trench cutoff, once consolidated under its own weight (usually within 6 months after placement), should be compatible with the compressibility of the surrounding ground to minimize differential movement of the dam and resultant stress concentrations in the embankment or its foundation (Ryan 1976 and Xanthakos 1979).

(b) Blowout Requirements. Once the slurry trench is installed, the dam has been constructed, and the reservoir filled, there is a substantial differential head acting on the slurry trench (see table 9-2 for typical values). Depending upon the characteristics of the backfill material and pervious foundation, the hydraulic gradient acting across the slurry trench may be sufficient to cause blowout or piping of backfill material into the surrounding pervious foundation. This is especially critical when the foundation contains openwork gravel where the piping process could result in the formation of channels and cavities that may breach the slurry wall. Based upon laboratory tests conducted on widely graded gravel containing no sand, the blowout gradient ranges from 25 to 35, depending on the properties of the backfill material (La Russo 1963 and Nash 1976). The factor of safety against blowout is

\[
F = \frac{i_{\text{allowable}}}{i_{\text{actual}}}
\]  

(9-9)

where

\[
F = \text{factor of safety against blowout}
\]

\[
i_{\text{allowable}} = \text{allowable hydraulic gradient from laboratory blowout tests}
\]

\[
i_{\text{actual}} = \text{actual hydraulic gradient existing on slurry trench}
\]

Substituting for the actual hydraulic gradient

\[
i_{\text{actual}} = \frac{\Delta h}{w}
\]  

(9-10)

where

\[
\Delta h = \text{maximum differential hydraulic head acting on the slurry trench}
\]

\[
w = \text{slurry trench width}
\]
and using a factor of safety of 3 and an allowable hydraulic gradient of 30 in equation 9-9 gives

\[ w = \frac{\Delta h}{10} \]  

(9-11)

If the pervious foundation contains openwork gravel, the width of soil-bentonite slurry trench required to prevent blowout failure may be estimated from equation 9-11. Further refinements on the trench width would require conducting laboratory blowout tests (as described by Xanthakos 1979).

(c) Permeability. For design purposes the permeability of the soil-bentonite slurry trench cutoff is based on the backfill material only (Xanthakos 1979). The permeability of the slurry trench is a function of both the filter cake that forms on the trench walls and the backfill material. The contribution of the filter cake and the backfill depends on the relative permeability and thickness of the two materials. The horizontal permeability of the soil-bentonite slurry trench is (D'Appolonia 1980)

\[ k = \frac{t_b}{t_b + \frac{2t_c}{k_b} + \frac{t_c}{k_c}} \]  

(9-12)

where

- \( k \) = permeability of slurry trench
- \( t_b \) = backfill thickness
- \( k_b \) = backfill permeability
- \( t_c \) = filter cake thickness
- \( k_c \) = filter cake permeability

The permeability of the backfill material can be determined in a laboratory permeability test (EM 1110-2-1906). The thickness of the backfill is selected in design (see figure 9-6). The ratio \( k_c/t_c \) is determined from the filter press test (American Petroleum Institute 1982) using various formation cake pressures as shown in figure 9-9a. For a range of practical applications, the ratio \( k_c/t_c \) varies from \( 5 \times 10^{-9} \)/sec to \( 25 \times 10^{-9} \)/sec as shown in figure 9-9b. Figure 9-10 shows the permeability of a soil-bentonite slurry trench cutoff wall 80 cm (about 2-1/2 ft) thick for various values of backfill permeability and ratios of \( k_c/t_c \). As shown in figure 9-10, the slurry trench
a. Schematic of filter press test apparatus

b. Relationship among filter cake permeability, filter cake formation pressure, and time

Figure 9-9. Determination of filter cake permeability (courtesy of American Society of Civil Engineers)
Figure 9-10. Permeability of a soil-bentonite slurry trench cutoff wall 80 cm thick for various values of backfill and filter cake permeability (courtesy of American Society of Civil Engineers)
permeability is controlled by the backfill when the backfill permeability is low and by the filter cake when the backfill permeability is high. Also, the slurry trench permeability has an upper limit of about $10^{-6}$ cm/sec even for very permeable backfill due to the thin low permeability filter cake (D'Appolonia 1980).

(d) Shear Strength. Soil-bentonite slurry trench cutoffs are difficult to sample because of their soft nature and very little data are available on the shear strength of soil-bentonite slurry trench backfill material. For design purposes, in conducting the stability analysis of the embankment and foundation, the shear strength of the backfill material is assumed to be zero. However, the shear strength of the backfill material does increase with time due to consolidation and thixotropy. At time of placement, the backfill material will stand on a slope ranging from 1V on 5H to 1V on 10H. This improves to about 1V on 2H with time (Ryan 1976 and D'Appolonia 1980). The results of shear strength tests (see table 9-5) on undisturbed samples taken from the soil-bentonite slurry trench at Saylorville Dam, Iowa, show that the undrained shear strength of the slurry backfill about a year after placement was 0.10 to 0.12 tons/sq ft (U. S. Army Engineer District, Rock Island 1978b).

(e) Compressibility. The compressibility of the soil-bentonite slurry trench backfill material depends primarily on the percentage of granular particles in the gradation as shown in figure 9-11. Low permeability and low compressibility are contradictory requirements because the plastic fines required for low permeability result in higher compressibility. Relatively low compressibility results when there is sufficient granular material in the backfill to allow grain-to-grain contact between the granular particles (D'Appolonia 1980).

(f) Mix Design. The gradation of the backfill for the soil-bentonite slurry trench is selected by conducting permeability, shear strength, and compressibility tests on a range of materials including soil to be excavated from the trench. Such a procedure was followed in the mix design for the backfill of the soil-bentonite slurry trench installed for remedial seepage control at Addicks Dam, Texas (U. S. Army Engineer District, Galveston 1977c). The allowable range set on the gradation of the backfill should produce a material which contains enough fines to reduce the seepage through the slurry trench cutoff to an acceptable level and sufficient coarse particles to approximate the strength and compressibility of the surrounding ground. If sufficient fines are not present in material excavated from the trench, borrow sources should be identified or alternatively a higher bentonite content specified for the backfill. If sufficient coarse particles are not present in material excavated from the trench, approved sources should be identified for obtaining natural sound, hard, durable sand and gravel. Crushed limestone, dolomite, or other crushed calcareous materials should not be used. The maximum particle size of the gravel shall be 3 in. and the material should be well graded.

(5) Cement-Bentonite Slurry Trench Cutoff.
Table 9-5. Summary of Shear Strength Data from Post-Construction Testing of Backfill Material from Soil-Bentonite Slurry Trench Cutoff at Saylorville Dam, Iowa (a)

<table>
<thead>
<tr>
<th>Boring No.</th>
<th>Station</th>
<th>Depth ft</th>
<th>Unified Soil Classification</th>
<th>Atterberg Limits</th>
<th>Dry Density lb/cu ft</th>
<th>Moisture Content %</th>
<th>Time Elapsed, months</th>
<th>Triaxial Compression Test (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>LL, PL, ML</td>
<td></td>
<td></td>
<td>Construction to Sampling</td>
<td>Sampling to Testing</td>
</tr>
<tr>
<td>STU-1</td>
<td>58+90</td>
<td>26.7-28.9</td>
<td>Gravelly clayey sand, SC</td>
<td>33, 14, 19</td>
<td>118.2</td>
<td>13.3</td>
<td>8.5</td>
<td>5</td>
</tr>
<tr>
<td>STU-1</td>
<td>58+90</td>
<td>39.2-41.4</td>
<td>Clayey sand, SC</td>
<td>33, 14, 19</td>
<td>108.5</td>
<td>17.5</td>
<td>8.5</td>
<td>5</td>
</tr>
<tr>
<td>STU-2</td>
<td>64+00</td>
<td>14.5-16.6</td>
<td>Clayey sand, SC</td>
<td>36, 15, 21</td>
<td>103.5</td>
<td>21.0</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>STU-1</td>
<td>58+90</td>
<td>49.2-51.4</td>
<td>Clayey sand, SC</td>
<td>41, 17, 24</td>
<td>107.9</td>
<td>18.9</td>
<td>8.5</td>
<td>19</td>
</tr>
<tr>
<td>STU-2</td>
<td>64+00</td>
<td>32.0-34.3</td>
<td>Clayey sand, SC</td>
<td>33, 14, 19</td>
<td>114.1</td>
<td>14.9</td>
<td>8</td>
<td>19</td>
</tr>
<tr>
<td>STU-6</td>
<td>82+90</td>
<td>16.0-18.1</td>
<td>Gravelly clayey sand, SC</td>
<td>34, 15, 19</td>
<td>120.1</td>
<td>16.9</td>
<td>11.5</td>
<td>7.5</td>
</tr>
<tr>
<td>STU-6</td>
<td>82+90</td>
<td>49.9-51.9</td>
<td>Gravelly clayey sand, SC</td>
<td>37, 15, 22</td>
<td>113.1</td>
<td>17.7</td>
<td>11.5</td>
<td>7.5</td>
</tr>
<tr>
<td>STU-7</td>
<td>87+80</td>
<td>17.0-19.0</td>
<td>Gravelly clayey sand, SC</td>
<td>32, 17, 15</td>
<td>113.5</td>
<td>17.6</td>
<td>11</td>
<td>6</td>
</tr>
<tr>
<td>STU-6</td>
<td>82+90</td>
<td>35.4-35.6</td>
<td>Sandy clay, CL</td>
<td>31, 18, 13</td>
<td>117.0</td>
<td>13.1</td>
<td>11.5</td>
<td>9.5</td>
</tr>
<tr>
<td>STU-7</td>
<td>87+80</td>
<td>26.6-28.8</td>
<td>Clayey sand, SC</td>
<td>28, 14, 14</td>
<td>112.9</td>
<td>19.2</td>
<td>11</td>
<td>11.5</td>
</tr>
</tbody>
</table>

(a) From U.S. Army Engineer District, Rock Island.
(b) Conducted on undisturbed soil specimens 5 in. in diameter by 11 in. high; unconsolidated-undrained (Q) and consolidated-undrained with pore pressure measurements (R) shear tests.
Figure 9-11. Relationship between fines content and compressibility of a soil-bentonite slurry trench backfill (courtesy of American Society of Civil Engineers)
(a) General. If backfill for the slurry trench is not available or is prohibitive in cost or if the cutoff is deep and the foundation is prone to failure during excavation, the cement-bentonite slurry trench cutoff may be more applicable (see table 9-3).

(b) Design Considerations. The primary design parameters are continuity, set time, resistance to hydraulic pressure, permeability, shear strength, and compressibility.

(c) Continuity. When cement-bentonite slurry trench cutoffs are constructed in panels rather than in a continuous trench, there is a possibility for unexcavated portions of the trench to remain between the panels. To prevent this the clamshell bucket is moved both vertically and horizontally throughout each slot at the completion of slot excavation. Also, when the connecting area between the initial and subsequent panels is excavated, a portion of the adjacent set panels is removed to ensure that all intervening soil has been excavated (Spooner et al. 1982).

(d) Set Time. The set time is important because of the construction technique employed. After the cement-bentonite slurry in the first set of panels has set, the areas between them can be excavated. A normal cement-bentonite mixture begins to set after a few hours and has a consistency similar to lard after 12 hours. The second day the cement-bentonite slurry can be walked on and final set is normally taken at 90 days (Ryan 1977).

(e) Resistance to Hydraulic Pressure. Once the slurry trench has been completed, the embankment constructed, and the reservoir filled, there is a substantial differential head acting on the slurry trench (see table 9-2 for typical values). The time between completion of the slurry trench and reservoir filling is generally sufficiently long (≥ 90 days) to allow the cement-bentonite slurry trench to develop its final set. The resistance of the cement-bentonite material to withstand gradients comparable to those which will exist in the field should be tested in the laboratory by subjecting intact specimens which have developed full set to hydraulic pressure and measuring the increase (if any) of permeability with time (Spooner et al. 1982 and Jefferis 1981).

(f) Permeability. Although there is some buildup of concentration near the sides of the cement-bentonite slurry trench, the cement-bentonite does not form a low permeability cake. The permeability of the slurry trench is a function of the concentrations of cement, bentonite, sand, and gravel (suspended during the excavation process) in the completed wall (Ryan 1977). The amount of sand and gravel in the cement-bentonite trench cutoff may range from 10 to 60 percent by dry weight, depending on the foundation material and method of construction, and generally increases with depth (Dank 1981). If the trench is excavated under a conventional bentonite slurry which is then replaced by a cement-bentonite slurry, the sand and gravel content will be low (10 to 18 percent was measured on undisturbed samples taken from the San Lorenzo Dam, El Salvador; Dank 1981). Alternatively, if the trench is excavated under a cement-bentonite slurry which is left in the trench to set up and form the cutoff, the sand and gravel content will be relatively high. Also, if the trench is excavated under a cement-bentonite slurry, the slurry loss into
the surrounding ground will be higher than normal and in some instances as great as 100 percent of the trench volume (Xanthakos 1979). For design purposes, specimens should be prepared from the cement-bentonite with varying percentages of sand and gravel, cured for 28 days under consolidation pressures existing in the field, and laboratory permeability tests conducted (EM 1110-2-1906).

(g) Shear Strength. Cement-bentonite slurry trench cutoffs are more easily sampled and tested than are soil-bentonite slurry walls. Also, specimens of cement-bentonite may be cast in the laboratory and tested. The results of shear strength tests (see table 9-6) on undisturbed samples taken from the cement-bentonite slurry trench at Tilden Tailings Project, Michigan, show the unconfined compressive strength about 6 months after placement increased with depth ranging from 0.88 to 1.43 tons/sq ft (Dank 1981).

(h) Compressibility. Very little data are available on the compressibility of cement-bentonite slurry trench material (Millet and Perez 1980). The compressibility should decrease with increase in cement to water ratio (provided the bentonite is fully hydrated with water prior to adding the cement) and with increase in sand content (once the concentration of suspended sand and gravel is sufficient to allow grain-to-grain contact between the granular materials).

(i) Mix Design. The cement-bentonite slurry trench mixture consists of water, bentonite, cement, set retarders as necessary, and sand and gravel entering the trench as a by-product of the excavation. The bentonite should be fully hydrated with water prior to adding the cement (Millet and Perez 1980). A retarder of the lignosulphite group may be added in small amounts (0.1 percent) to delay the initial set to avoid hardening of the mix in the panel before the excavation is completed (Xanthakos 1979). When low permeability is required, the bentonite content of the slurry should be increased (in the range from 3 to 6 percent by dry weight). Increased sand and gravel in the slurry will result in an increase in permeability (Dank 1981). The cement, sand, and gravel content are the chief factors in controlling the strength and deformability characteristics of the slurry mix (see table 9-6 and figure 9-12). Generally, the higher the cement to water ratio, the higher the strength, and more brittle (lower failure strain) the cement-bentonite slurry mix (Millet and Perez 1980). By varying the bentonite and cement quantities, flexibility can be designed into the cement-bentonite slurry trench cutoff. This is especially important if the dam is located at a site where strong earthquake shocks are likely. The cement-bentonite slurry mix proportions should be selected by conducting permeability, shear strength, and compressibility tests on a range of materials including soil to be excavated from the trench. Varying proportions of water, bentonite, cement, sand and gravel (representing aggregate entering the trench during the excavation process) should be tested to select a design mix which will reduce the seepage through the slurry trench cutoff to an acceptable level and approximate the strength and compressibility of the surrounding ground.

(6) Failure Mechanisms of Cutoffs
### Table 9-6. Summary of Shear Strength Data from Post-Construction Testing of Cement-Bentonite Slurry Trench Cutoff at Tilden Tailings Project, Michigan (a)

<table>
<thead>
<tr>
<th>Boring No.</th>
<th>Depth (ft)</th>
<th>Dry Density (lb/cu ft)</th>
<th>Sand Content By Volume (%)</th>
<th>Moisture Content (%)</th>
<th>Time Elapsed Construction to Sampling months</th>
<th>Unconfined Compressive Strength (tons/sq ft)</th>
<th>Triaxial Compression Test (b)</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6.0</td>
<td>26.5</td>
<td>2.5</td>
<td>271.6</td>
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<tr>
<td>1</td>
<td>10.7</td>
<td>29.4</td>
<td>11.6</td>
<td>151.4</td>
<td>6</td>
<td>0.94</td>
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<tr>
<td>1</td>
<td>15.9</td>
<td>32.1</td>
<td>13.2</td>
<td>141.0</td>
<td>6</td>
<td>0.97</td>
<td></td>
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<tr>
<td>1</td>
<td>21.0</td>
<td>37.0</td>
<td>11.4</td>
<td>138.8</td>
<td>6</td>
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<td>1</td>
<td>31.0</td>
<td>37.5</td>
<td>6.2</td>
<td>129.0</td>
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<td></td>
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<tr>
<td>1</td>
<td>41.0</td>
<td>40.0</td>
<td>--</td>
<td>115.8</td>
<td>6</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>2</td>
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<td>0.0</td>
<td>236.0</td>
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<td>2</td>
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<td>6.2</td>
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<td>6</td>
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<td>11.4</td>
<td>134.5</td>
<td>6</td>
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<td>13.4</td>
<td>117.1</td>
<td>6</td>
<td>1.30</td>
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</table>

(a) From Northwestern University

(b) Conducted on undisturbed soil specimens; consolidated-drained (S) shear test
Figure 9-12. Strength and deformation characteristics of cement-bentonite slurries (courtesy of American Society of Engineers)
(a) Introduction. Several mechanisms can affect the functioning of slurry walls and cause failure. Failure may occur during excavation of the trench, upon first filling of the reservoir with the resulting rise in differential head acting across the slurry wall, or at some future time due to adverse chemical substances in the soil and ground water. Specific failure mechanisms include trench collapse, gaps (or windows) in the slurry wall, inadequate aquiclude key-in, blowout or piping of backfill material into the surrounding pervious foundation, and chemical destruction of the slurry wall (Spooner et al. 1982).

(b) Trench Collapse. Trench collapse is caused by instability of the trench walls during excavation and before backfilling (for soil-bentonite slurry trench) or setup (for cement-bentonite slurry trench). Causes of trench collapse include failure to maintain the minimum differential head between the top of the slurry and the top of the ground water and/or too low unit weight of slurry (see figure 9-7). Drop in the slurry level in the trench may be caused by contact with gravel, fissures, etc., during excavation, while rise in the ground-water level may be caused by surface runoff into cracks adjacent to the trench, particularly following heavy rainfall. Too low unit weight of the slurry may be caused by the cessation of agitation by excavation equipment over the week end (Spooner et al. 1982). Such a set of circumstances contributed to the collapse of a portion of one wall of a soil-cement bentonite slurry trench at Duncan Dam in Canada (Duguid et al. 1971).

(c) Gaps (or Windows) in the Slurry Wall. Trench collapse or improper placement of backfill can create gaps (or windows) which result in zones of higher permeability as well as variations in wall thickness and strength (Spooner et al. 1982). The continuity of the trench should be tested before backfilling by passing the bucket or clamshell of the excavating tool vertically and horizontally along each segment of the trench. As mentioned previously, for soil-bentonite slurry trenches irregularities in the backfill slope are indications that pockets of clean material (slurry not mixed in properly or was washed out) or slurry were trapped in the backfill or that the backfill does not contain sufficient sand or coarse material.

(d) Inadequate Aquiclude Key-In. As discussed previously, inadequate aquiclude key-in will permit seepage under the cutoff. Inadequate key-in can result from variations in trench depth, insufficient aquiclude penetration, trench collapse, or the presence of boulders (Spooner et al. 1982).

(e) Blowout or Piping of Backfill Material. As mentioned previously, blowout or piping of backfill material into the surrounding pervious foundation is especially critical for soil-bentonite slurry trenches when the foundation contains openwork gravel. The required width of the slurry trench to prevent blowout (factor of safety of 3) in openwork gravel may be estimated from Equation 9-11.

(f) Chemical Destruction of the Slurry Wall. Chemical substances in the foundation soil and ground water can affect the durability of the slurry wall once it is constructed. If salt water is present in the construction area, appapulgite may be used instead of bentonite. Permeation of a soil-bentonite or cement-bentonite slurry wall by polluted ground water generally
leads to increased permeability. The bentonite may become entrained in the solution and the cement may become slurry solubilized as a solution channel is created through the slurry wall and into the foundation. Chemicals may also prevent the slurry from forming an adequate filter cake along the sides of the soil-bentonite slurry trench (Spooner et al. 1982). Where polluted ground water is present, long-term permeability tests should be conducted using the specific soil-bentonite backfill materials or cement-bentonite mix from the site permeated by the actual pollutant in designing the slurry trench cutoff (Spooner et al. 1982 and D'Appolonia 1980).

(7) Instrumentation and Monitoring.

(a) Introduction. Whenever a slurry trench is used for control of underseepage, the initial filling of the reservoir must be controlled and instrumentation monitored to determine if the slurry trench is performing as planned. If the slurry trench cutoff is ineffective, remedial seepage control measures must be installed prior to further raising of the reservoir pool (EM 1110-2-2300).

(b) Parameters of Interest. There are two parameters of interest with regard to slurry trench cutoffs for control of underseepage. These are the drop in piezometric head from upstream to downstream across the trench during reservoir filling and the differential settlement between the top of the slurry trench and the overlying compacted embankment material.

(c) Efficiency of Slurry Trench Cutoff. To evaluate the head efficiency (see equation 2) of the slurry trench cutoff, the head loss is determined between points immediately upstream and downstream of the slurry trench cutoff wall at its junction with the base of the dam. The head loss is established from piezometer readings taken during construction, before and during initial filling of the reservoir, and subsequently as frequently as necessary to determine changes that are occurring and to assess their implications with respect to safety of the dam (see Chapter 13). Equal numbers of piezometers are normally placed on each side of the slurry trench cutoff. Piezometers should be installed at two or more locations along the length of the slurry trench depending upon the foundation conditions at the site. Pneumatic piezometers installed upstream and downstream of the soil-bentonite slurry trench at West Point Dam, Alabama and Georgia, showed a near-horizontal piezometric surface existed across the trench prior to filling the reservoir. Piezometer readings taken after reservoir filling indicated a drop in piezometer head from upstream to downstream across the slurry trench, confirming the effectiveness of the cutoff (U. S. Army Engineer District, Savannah 1979). Open-tube piezometers installed upstream and downstream of the soil-bentonite slurry trench at Addicka Dam, Texas, indicated a drop in piezometric head across the slurry trench (U. S. Army Engineer District, Galveston 1983).

(d) Differential Settlement. The differential settlement between the top of the slurry trench and the overlying compacted embankment material is important because a separation of materials in this region could result in piping at the interface between the embankment and the foundation. Settlement plates placed on top of the soil-cement bentonite slurry trench at West Point Dam, Alabama and Georgia, indicated a uniform total settlement of approximately
0.5 ft throughout the trench. Excavation of a portion of the trench prior to filling the reservoir showed no void between the slurry trench backfill and the overlying compacted fill (U. S. Army Engineer District, Savannah 1968 and 1979).

e. Concrete Wall.

(1) Introduction. When the depth of the pervious foundation is excessive (< 150 ft) and/or the foundation contains cobbles, boulders, or cavernous limestone, the concrete cutoff wall may be an effective method for control of underseepage. Using this method, a cast-in-place continuous concrete wall is constructed by tremie placement of concrete in a bentonite slurry supported trench. Two general types of concrete cutoff walls, the panel wall and the element wall have been used, as shown in figures 9-13 and 9-14, respectively. Since the wall in its simpler structural form is a rigid diaphragm, earthquakes could cause its rupture; therefore, cutoff walls should not be used at a site where strong earthquake shocks are likely (U. S. Army Engineer District, Pittsburgh 1965).

(2) History of Use. Conventional (excavated without bentonite slurry) concrete cutoff walls were widely used prior to 1925. Since they require about the same excavation and dewatering as compacted backfill trenches and the wall itself is far more expensive than compacted soil, the popularity of conventional concrete cutoff walls has declined (Sowers 1962 and Sherard et al. 1963). The method of excavating trenches supported by bentonite for the construction of cast-in-place concrete cutoff walls was used for the first time in 1951 at the Volturno-Garigliano hydroelectric plant on the Volturno River at Venafro, near Naples, Italy (Veder 1963, Veder 1975 and Franke 1954). Since the 1950's, concrete cutoff walls constructed by tremie placement of concrete in a bentonite slurry supported trench have been used for projects throughout the world. The deepest concrete cutoff wall to date was constructed at Manicouagan 3 Dam in Quebec, Canada, in 1972, where two parallel concrete walls, 2 ft thick and 10 ft apart, extended 430 ft deep (Anonymous 1972). A comparison of concrete cutoff walls constructed at Corps of Engineers dams is given in table 9-7.

(3) Sequence of Construction and Location of Wall. Normally the embankment is constructed first, followed by the concrete cutoff wall located upstream of the toe of the dam as was done at Kinzua Dam (formerly Allegheny Dam) and tied into the core of the dam with an impervious blanket (U. S. Army Engineer District, Pittsburgh 1965). The upstream location minimizes the possibility of compressive failure of the concrete cutoff wall due to negative skin friction as the foundation settles under the weight of the embankment (as would occur if the cutoff wall is located under the center of the dam). Constructing the embankment first, followed by the concrete cutoff wall, minimizes the possibility of rupture of the concrete cutoff wall due to horizontal movement of the foundation as the embankment is constructed. For remedial seepage control of existing dams (see Chapter 12) where it is not practical to draw down the reservoir and primary consolidation of the foundation has been complete, a central location for the concrete cutoff wall may be feasible.
Figure 9-13. Construction procedure for concrete cutoff wall at Kinzua Dam (formerly Allegheny Dam), Pennsylvania (after U. S. Army Engineer District, Pittsburgh).

(4) Design Considerations. The primary design parameters are permeability, strength, and compressibility. The permeability is usually sufficiently low ($10^{-10}$ cm/sec for water-cement ratio of 0.6) to reduce the seepage through the concrete cutoff wall to an acceptable value (Xanthakos 1979). The concrete cutoff wall is generally stronger (>3,000 psi compressive strength) than the surrounding foundation soil and introduces a heterogeneous zone (in the form of a rigid diaphragm) in the foundation. The compressibility of the concrete cutoff wall is sufficiently low that the wall is essentially rigid with respect to the surrounding foundation soil (Xanthakos 1979).

(a) Permeability. For workable concrete mixes used in concrete cutoff walls (see table 9-8), the permeability increases rapidly for water-cement ratios higher than 0.5. For a concrete mix with maximum coarse aggregate size

(1) The workability of the concrete, discussed under Mix Design, is of primary importance with respect to tremie placement of the concrete.
a. Excavation procedure for primary and secondary elements

b. Interlocking of primary and secondary elements

Figure 9-14. Construction procedure for concrete cutoff wall at Wolf Creek Dam, Kentucky (courtesy of American Society of Civil Engineers)
### Table 9-7. Comparison of Corps of Engineers Concrete Cutoff Walls

<table>
<thead>
<tr>
<th>Project</th>
<th>Location</th>
<th>Date Constructed</th>
<th>Embankment-Foundation Material</th>
<th>Max. Differential Head</th>
<th>Max. Depth</th>
<th>Location</th>
<th>Max. (a) Deviation from Vertical</th>
<th>Max. Head Cutoff Width</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinzua (formerly Allegheny) Dam</td>
<td>Allegheny River Pennsylvania</td>
<td>1964</td>
<td>Silts, sands and gravels, boulders</td>
<td>140</td>
<td>2.5</td>
<td>185</td>
<td>Upstream of toe of dam</td>
<td>1:370</td>
<td>56.0 U. S. Army Engineer District, Pittsburgh 1965</td>
</tr>
<tr>
<td>Wolf Creek Dam (b)</td>
<td>Cumberland River Kentucky</td>
<td>1978</td>
<td>Embankment (200 ft); cavernous limestone (100 ft)</td>
<td>187</td>
<td>2.0</td>
<td>300</td>
<td>Center of dam</td>
<td>1:600</td>
<td>93.5 U. S. Army Engineer District, Nashville 1975; Fetzer 1979</td>
</tr>
<tr>
<td>Mill Creek Dam</td>
<td>Mill Creek Washington</td>
<td>1981</td>
<td>Silt overlying conglomerate</td>
<td>51</td>
<td>2.0</td>
<td>170</td>
<td>Upstream toe of dam</td>
<td>1:100</td>
<td>25.5 U. S. Army Engineer District, Walla Walla 1980a,b</td>
</tr>
<tr>
<td>Clemson Lower, Diversion Dam</td>
<td>Hartwell Lake Georgia and South Carolina</td>
<td>1982</td>
<td>Embankment (50 ft); silty sand and sand (35 ft)</td>
<td>44</td>
<td>2.0</td>
<td>85</td>
<td>Center of dam</td>
<td>1:100</td>
<td>22.0 U. S. Army Engineer District, Savannah 1981</td>
</tr>
</tbody>
</table>

(a) Deviation from vertical in any direction at any depth.

(b) Concrete cutoff installed as remedial seepage control for existing dam (see Chapter 12).
### Table 9-8. Comparison of Concrete Mix Proportions for Corps of Engineers Concrete Cutoff Walls

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinzua (formerly Allegheny Dam)</td>
<td>564</td>
<td>0</td>
<td>292</td>
<td>0.52</td>
<td>1,652</td>
<td>1,345</td>
<td>3,000</td>
<td>7</td>
<td>3,000</td>
<td>3,843</td>
</tr>
<tr>
<td>Wolf Creek Dam</td>
<td>564</td>
<td>123</td>
<td>275</td>
<td>0.40</td>
<td>1,619</td>
<td>1,369</td>
<td>6.5-7.5</td>
<td>3,000</td>
<td>4,685</td>
<td></td>
</tr>
<tr>
<td>Mill Creek Dam</td>
<td>299</td>
<td>134(b)</td>
<td>260</td>
<td>0.60</td>
<td>1,628</td>
<td>1,587</td>
<td>8</td>
<td>1,500</td>
<td>1,970</td>
<td></td>
</tr>
<tr>
<td>Clemson Lower Diversion Dam</td>
<td>507</td>
<td>110</td>
<td>334</td>
<td>0.54</td>
<td>1,540</td>
<td>1,310</td>
<td>60.9.0</td>
<td>1,300</td>
<td>--</td>
<td></td>
</tr>
</tbody>
</table>

(a) Weights are saturated surface dry.
(b) Fly ash percentage replaced by solid volume was raised from 35 to 45 before wall was completed.
of 3/4 in. and a water cement ratio of 0.6, the permeability is usually lower than \(10^{-10} \text{ cm/sec}\) (Xanthakos 1979). The permeability of a concrete cutoff wall is influenced by cracks in the finished structure and/or by void spaces left in the concrete as a result of honeycombing or segregation (see Equation 9-4 and figure 9-5). The joints between panels are not completely impermeable but the penetration of bentonite slurry into the soil in the immediate vicinity of the joint usually keeps the flow of water very small (Hanna 1978). Measured head efficiency for concrete cutoff walls from piezometric data generally exceeds 90 percent (Telling, Menzies, and Simons 1978b). At Kinzua Dam (formerly Allegheny Dam), the measured head efficiency was 100 percent, i.e., the head just downstream of the concrete cutoff wall was of the magnitude established by vertical seepage through the upstream connecting blanket (Fuquay 1968).

(b) Strength. The compressive strength for concrete cutoff walls is generally specified to exceed 3,000 lb/sq in. (see table 9-8). Therefore, the concrete cutoff wall is generally stronger than the surrounding foundation soil. The most important factor influencing the strength of the concrete is the water-cement ratio. The concrete's fluidity, i.e., ability to travel through the tremie and fill the excavation, also depends upon the water-cement ratio. Too low a water-cement ratio would decrease flowability and increase compressive strength. Too high a water-cement ratio would promote segregation. A good balance is achieved with a water-cement ratio near 0.5 which results in a 28-day compressive strength exceeding 3,000 lb/sq in. (see table 9-8). Cement continues to hydrate and concrete continues to increase in compressive strength, at a decreasing rate, long after 28 days (Winter and Nilson 1979).

(c) Compressibility. The concrete cutoff wall is essentially rigid and has low compressibility compared to the surrounding foundation soil. The modulus of elasticity for concrete cutoff walls may be approximated from (Winter and Nilson 1979)

\[
E_c = 33W^{3/2}F_c^{1/2}
\]  

(9-13)

where

- \(E_c\) = modulus of elasticity in lb/sq in.
- \(W\) = unit weight of concrete in lb/cu ft
- \(F_c\) = compressive strength of concrete in lb/sq in.

(5) Mix Design. In addition to strength, workability is an important requirement for the concrete mix. The mix must not segregate during placement. Too high a water-cement ratio or too low a cement content (with a good water-cement ratio) will tend to segregate. Natural well rounded aggregate increases flowability and allows the use of less cement than an angular
manufactured aggregate. Since the concrete is poured into the trench through tremie pipes and displaces the bentonite slurry from the bottom of the excavation upward, the concrete must have a consistency such that it will flow under gravity and resist mixing with the bentonite slurry. Admixtures may be used as required to develop the desired concrete mix characteristics. Fly ash is often used to improve workability and to reduce heat generation. The unique problems inherent at each project require studies to develop an adequate concrete mix (Holland and Turner 1980). Some typical concrete mixes used in Corps of Engineers concrete cutoff walls are given in table 9-8. The placement techniques used for the concrete are of equal importance in assuring a satisfactory concrete cutoff wall.

(6) Excavation and Placement of Concrete. Temporary guide walls are constructed at the ground surface to guide the alignment of the trench and support the top of the excavation. Typically, a cross section, 1 ft wide and 3 ft deep, is sufficient for most concrete cutoff walls. In order to ensure continuity between panels and provide a watertight joint to prevent leakage, an appropriate tolerance is placed on the maximum deviation from the vertical (see table 9-7). The same general requirements apply to the slurry used to keep the trench open for concrete cutoffs. As stated previously, two general types of concrete cutoff walls, the panel wall, and the element wall have been used. The panel wall is best suited for poorly consolidated materials and soft rock can be installed to about a 200-ft depth. The element wall has the advantage of greater depth (430 ft deep at Manicouagan 3 Dam in Quebec, Canada), better control of verticality, the ability to penetrate hard rock using chisels and/or nested percussion drills, and the protection of the embankment with casing when used for remedial seepage control. However, the element wall is more costly and has a slower placement rate than the panel wall. Both types of concrete cutoff walls open short horizontal sections of the embankment and/or foundation at a time, which limits the area for potential failure to a segment that can be controlled or repaired without risking catastrophic failure of the project. The concrete cutoff wall penetrates the zone(s) of seepage with a rigid, impermeability barrier capable of withstanding high head differentials across cavities with no lateral support. The concrete must be placed at considerable depth through bentonite slurry in a continuous operation with as little contamination, honeycomb, or segregation as possible. The bottom of the excavation must be cleaned so that a good seal can be obtained at grade. Fresh bentonite slurry is circulated through the excavation to assist in the cleaning and lower the density of the slurry to allow the concrete to displace the slurry easier once placement begins. The tremie procedure used to place the concrete is straightforward in theory and yet often in practice causes more problems with the final quality of the concrete cutoff wall than any other factor. The tremie system consists of a hopper, tremie pipe, and a crane or other lifting equipment to support the apparatus. The hopper should be funnel shaped and have a minimum capacity of 0.5 cu yd. The size of the tremie pipe depends upon the size of aggregate used in the concrete mix. For 3/4-in. maximum diameter coarse aggregate, a 10-in.-diam tremie pipe
should be used.\(^{(1)}\) The dry tremie is placed in the hole with a metal plate and rubber gasket wired to the end of the tremie. The tremie pipe is lifted, breaking the wires and allowing the concrete flow to begin. Concrete is added to the hopper at a uniform rate to minimize free fall to the surface in the pipe and obtain a continuous flow. The tremie apparatus is lifted during placement at a rate that will maintain the bottom of the pipe submerged in fresh concrete at all times and produce the flattest surface slope of concrete that can practically be achieved. The flow rate (foot of height per hour) and surface slope of the concrete shall be continuously measured during placement with the use of a sounding line. A sufficient number of tremies should be provided so that the concrete does not have to flow horizontally from a tremie more than 10 ft. As soon as practical, core borings should be taken in selected panels through the center of the cutoff wall to observe the quality of the final project. Unacceptable zones of concrete such as honeycombed zones, segregated zones, or uncemented zones found within the cored panels or elements should be repaired or removed and replaced. One means of minimizing such problems at the start of a job is to require a test section in a noncritical area to allow changes in the construction procedure to be made early in the project (Hallford 1983; Holland and Turner 1980; and Gerwick, Holland, and Komendant 1981).

(7) Treatment at Top of Concrete Cutoff Wall. As mentioned previously, normally the concrete cutoff wall is located under or near the upstream toe of the dam and tied into the core of the dam with an impervious blanket. If a central location for the concrete cutoff wall is dictated by other factors, the connection detail between the top of the concrete cutoff wall and the core of the dam is very important. Generally, the concrete cutoff wall extends upward into the core such that, the hydraulic gradient at the surface of the contact does not exceed 4 (Wilson and Marsal 1979). Various precautions (see figure 9-15) have been taken to prevent the top of the concrete cutoff wall from punching into the core of the dam and causing the core to crack as the foundation settles on either side of the rigid cutoff wall under the weight of the embankment. The bentonite used at the connection between the concrete cutoff wall and the core of the dam (see figure 9-15) is intended to create a soft zone to accommodate differential vertical settlements of the core around the concrete cutoff wall. Also, saturation of the bentonite is intended to produce swelling which will provide for a bond between the core and the concrete cutoff wall to prevent seepage (Radukic 1979).

(8) Failure Mechanisms of Concrete Cutoff Walls. Several mechanisms can affect the functioning of concrete cutoff walls and cause failure. As mentioned previously, the wall in its simpler structural form is a rigid diaphragm and earthquakes could cause its rupture. For this reason concrete cutoff walls should not be used at a site where strong earthquake shocks are

\(^{(1)}\) At Wolf Creek Dam concrete problems (areas of segregated sand or coarse aggregate, voids, zones of trapped laitance, and honeycombed concrete) occurred for tremie-placed 26-in.-diam cased primary elements. This must be considered in future projects which involve tremie-placed elements of small cross-sectional areas (Holland and Turner 1980).
a. Forked connection  
b. Plastic impervious cap  
c. Piston connection  
d. Double wall connection  

Figure 9-15. Connections between concrete cutoff wall and core of dam (courtesy of ICOS)
likely. Concrete cutoff walls located under or near the toe of the dam are subject to possible rupture from horizontal movements of the foundation soil during embankment construction. This effect can be minimized by constructing the dam embankment prior to the concrete cutoff wall. As mentioned previously, concrete cutoff walls located under the center of the dam are subject to possible compressive failure due to negative skin friction as the foundation settles under the weight of the embankment. The probability of this occurring would depend upon the magnitude of the negative skin friction developed at the interface between the concrete cutoff wall and the foundation soil and the stress-strain characteristics of the concrete cutoff wall. Also, as previously mentioned, a centrally located concrete cutoff wall may punch into and crack the overlying core material unless an adequate connection is provided between the concrete cutoff wall and the core of the dam.

(9) Instrumentation and Monitoring. Whenever a concrete cutoff wall is used for control of underseepage, the initial filling of the reservoir must be controlled and instrumentation monitored to determine if the concrete cutoff wall is performing as planned. If the concrete cutoff wall is ineffective, remedial seepage control measures must be installed prior to further raising the reservoir pool. When the embankment is constructed first, followed by the concrete cutoff wall located upstream of the toe of the dam, as was done at Kinzua (formerly Allegheny Dam), the parameters of interest are the drop in piezometric head from upstream to downstream across the concrete cutoff wall, differential vertical settlement between the upstream impervious blanket and the top of the concrete cutoff wall, and vertical and horizontal movement of the concrete cutoff wall due to reservoir filling. If a central location for the concrete cutoff wall is dictated by others factors, the parameters of interest are the drop in piezometric head from upstream to downstream across the cutoff wall, differential vertical settlement between the core of the dam and the top of the concrete cutoff wall, and vertical and horizontal movement of the concrete cutoff wall due to construction of the embankment and reservoir filling. Instrumentation data should be obtained during construction, before and during initial filling of the reservoir, and subsequently as frequently as necessary to determine changes that are occurring and to assess their implications with respect to the safety of the dam (see Chapter 13). The head efficiency for concrete cutoff walls is evaluated in the same manner as described previously for slurry trench cutoffs. As previously mentioned, measured head efficiency for concrete cutoff walls generally exceeds 90 percent.

f. Steel Sheetpiling.

(1) Introduction. Steel sheetpiling is rolled steel members with interlocking joints along their edges. Sheetpiling is produced in straight web, arch web, and Z sections in a graduated series of weights joined by interlocks to form a continuous cutoff wall as shown in figure 9-16. Steel sheetpiling is not recommended for use as a cutoff to prevent underseepage beneath dams due to the low head efficiency. Steel sheetpiling is frequently used in conjunction with concrete flood control and navigation structures to confine the foundation soil to prevent it from piping out from under the structure (EM 1110-2-2300 and Greer, Moorhouse, and Millet 1969).
a. Sections

b. Interlocking of sections

Figure 9-16. Steel sheetpiling installation (from U. S. Army Engineer Waterways Experiment Station)
(2) History of use. Steel sheetpiling was first used by the Corps of Engineers to prevent underseepage at Fort Peck Dam, Montana (U. S. Army Engineer District, Omaha 1982). The steel sheetpiling, driven to Bearpaw shale bedrock with the aid of hydraulic spade jetting, reached a maximum depth of 163 ft in the valley section (see table 9-9). An original plan to force grout into the interlocks of the steel sheetpiling was abandoned during construction as impractical. Steel sheetpiling was used as an extra factor to prevent piping of foundation soils at Garrison Dam, North Dakota (U. S. Army Engineer District, Omaha 1964). At Garrison Dam, underseepage control was provided for by an upstream blanket and relief wells and the contribution of the steel sheetpiling to reduction of underseepage was neglected in the design of the relief wells. Steel sheetpiling and an upstream blanket were installed for control underseepage at Oahe Dam, South Dakota. Relief wells were installed for remedial seepage control to provide relief of excess hydrostatic pressures developed by underseepage (U. S. Army Engineer District, Omaha 1961).

(3) Efficiency of Steel Sheetpiling Cutoffs. The efficiency of steel sheetpiling cutoffs is dependent upon proper penetration into an impervious stratum and the condition of the sheeting elements after driving. When the foundation material is dense or contains boulders which may result in ripping of the sheeting or damage to the interlocks (see figure 9-17), the efficiency will be reduced (Guertin and McTigue 1982). Theoretical studies indicate that very small openings in the sheeting (< 1 percent of the total area) will cause a substantial reduction in the cutoff efficiency (from 100 to 10 percent efficiency) as shown in figure 9-18 (Ambraseys 1963). The measured head efficiency for steel sheetpiling cutoffs installed at Corps of Engineers dams is given in table 9-9. The effectiveness of the steel sheetpiling is initially low, only 12 to 18 percent of the total head was lost across the cutoff as shown in table 9-9. With time, the head loss across the steel sheetpiling increased to as much as 50 percent of the total head. This increase in effectiveness is attributed to migration of fines and corrosion in the interlocks and reservoir siltation near the dam.

9-5. **Upstream Impervious Blanket.**

   a. **Introduction.** When a complete cutoff is not required or is too costly, an upstream impervious blanket tied into the impervious core of the dam may be used to minimize underseepage. Upstream impervious blankets should not be used when the reservoir head exceeds 200 ft because the hydraulic gradient acting across the blanket may result in piping and serious leakage. Downstream underseepage control measures (relief wells or toe trench drains) are generally required for use with upstream blankets to control underseepage and/or prevent excessive uplift pressures and piping through the foundation. Upstream impervious blankets are used in some cases to reinforce thin spots in natural blankets. Effectiveness of upstream impervious blankets depends upon their length, thickness, and vertical permeability, and on the stratification and permeability of soils on which they are placed (EM 1110-2-2300, Barron 1977 and Thomas 1976).

   (1) The blanket may be impervious or semipervious (leaks in the vertical direction).
Table 9-9. Comparison of Corps of Engineers Steel Sheetpiling Cutoff Walls

<table>
<thead>
<tr>
<th>Project</th>
<th>Location</th>
<th>Date Constructed</th>
<th>Foundation Materials</th>
<th>Max Depth Ft</th>
<th>Location</th>
<th>Purpose</th>
<th>Head Efficiency of Cutoff, %</th>
<th>Initial</th>
<th>With Time</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ft. Peck Dam</td>
<td>Missouri River, Montana</td>
<td>1940</td>
<td>Gravels, sands, and silts</td>
<td>163</td>
<td>Center of dam</td>
<td>Prevent underseepage</td>
<td>12</td>
<td>30(b)</td>
<td></td>
<td>Lane and Wohlt 1961</td>
</tr>
<tr>
<td>Garrison Dam</td>
<td>Missouri River, North Dakota</td>
<td>1954</td>
<td>Sand with gravel lenses</td>
<td>110</td>
<td>Upstream portion of dam</td>
<td>Prevent piping</td>
<td>18</td>
<td>50(b)</td>
<td></td>
<td>U. S. Army Engineer District, Omaha 1964</td>
</tr>
<tr>
<td>Oahe Dam</td>
<td>Missouri River, South Dakota</td>
<td>1958</td>
<td>Sands, gravels, and silts</td>
<td>75</td>
<td>Upstream portion of dam</td>
<td>Prevent piping</td>
<td>16</td>
<td>22(c)</td>
<td></td>
<td>U. S. Army Engineer District, Omaha 1961</td>
</tr>
</tbody>
</table>

(a) After 17 years (relief wells were installed in 1962 as remedial underseepage control)
(b) After 10 years (upstream blanket and relief wells were constructed prior to impoundment)
(c) After 4 years (upstream blanket constructed prior to impoundment and relief wells installed in 1962 as remedial underseepage control)
Figure 9-17. Sources of leakage associated with steel sheetpile cutoffs (from U. S. Department of Transportation\textsuperscript{41})
b. Design Considerations. In alluvial valleys, frequently soils consist of fine-grained top stratum of clay, silt, and silty or clayey sand underlain by a pervious substratum of sand and gravel. As stated previously, the top stratum or blanket may be impervious or semipervious (leaks in the vertical direction). The substratum aquifer or pervious foundation is generally anisotropic with respect to permeability so the flow is horizontal. For this condition, shown in figure 9-19, the basic assumptions for the design of upstream impervious blankets are:

(1) Flow through the blanket is vertical.

(2) Flow through the pervious foundation is horizontal.

(3) All flows are laminar and steady state.
a. Continuous blanket and aquifer

- \( L_1 \) = Effective length of upstream natural blanket
- \( L_2 \) = Length of embankment base
- \( L_3 \) = Effective length of downstream natural blanket
- \( L_0 \) = Length of discontinuous upstream blanket
- \( h \) = Net head to dissipate
- \( Z \) = Thickness of natural blanket
- \( d \) = Thickness of aquifer
- \( k_b \) = Permeability coefficient of blanket
- \( k_f \) = Permeability coefficient of aquifer
- \( \gamma_{sub} \) = Submerged unit weight of blanket
- \( h_o \) = Pressure head under blanket at downstream toe of dam
- \( h_C \) = Critical head under blanket at downstream toe of dam
- \( F_h \) = Factor of safety relative to heaving at downstream toe
- \( \gamma_w \) = Unit weight of water (63.4 pcf)
- \( q_f \) = Rate of discharge through aquifer with unit length normal to the section

Figure 9-19. Upstream impervious blanket (from U. S. Department of Agriculture)
(4) The dam (or core of a zoned embankment) is impervious.

(5) Both the blanket and substratum have a constant thickness and are horizontal.

When the top stratum or pervious foundation consists of several layers of different soils, they must be transformed into a single stratum with an effective thickness and permeability (see procedure given in U. S. Army Engineer Waterways Experiment Station 1956a). For the upstream impervious blanket shown in figure 9-19, the effective length of the upstream blanket is

\[ L_1 = \sqrt{\left(\frac{k_f}{k_{bR}}\right)Z_{bR}d} \]  \hspace{1cm} (9-14)

where

\[ L_1 = \text{effective length of upstream blanket} \]
\[ k_f = \text{horizontal permeability of pervious foundation} \]
\[ k_{bR} = \text{vertical permeability of upstream blanket} \]
\[ Z_{bR} = \text{thickness of upstream blanket} \]
\[ d = \text{thickness of pervious foundation} \]

The effective length of the downstream blanket is

\[ L_3 = \sqrt{\left(\frac{k_f}{k_{bL}}\right)Z_{bL}d} \]  \hspace{1cm} (9-15)

where

\[ L_3 = \text{effective length of downstream blanket} \]
\[ k_{bL} = \text{vertical permeability of downstream blanket} \]
\[ Z_{bL} = \text{thickness of downstream blanket} \]

Upstream blankets should be designed so that under maximum reservoir conditions the pressure head under the blanket at the downstream toe of the dam and the rate of discharge through the pervious foundation are acceptable. The pressure head under the blanket at the downstream toe of the dam (see figure 9-19) is
\[ h_o = \frac{h L_3}{L_1 + L_2 + L_3} \quad (9-16) \]

where

- \( h_o \) = pressure head under the blanket at the downstream toe of the dam
- \( h \) = net head to dissipate
- \( L_2 \) = length of impervious core or dam base

The critical pressure head under the blanket at the downstream toe of the dam is

\[ h_c = \frac{2b L \gamma_{sub}}{\gamma_w} \quad (9-17) \]

where

- \( h_c \) = critical pressure head under the blanket at the downstream toe of the dam
- \( \gamma_{sub} \) = submerged unit weight of downstream blanket soil
- \( \gamma_w \) = unit weight of water

The factor of safety against uplift or heaving at the downstream toe of the dam is

\[ F_h = \frac{h_c}{h_o} \quad (9-18) \]

where \( F_h \) is the factor of safety against uplift or heaving at the downstream toe of the dam. Generally dams are designed without relying upon natural downstream blankets because it is difficult to assure the continuity and the existence of the blanket throughout the life of the structure. Also, downstream seepage control measures (relief wells or trench drains) are generally used with upstream blankets to reduce uplift or heaving at the downstream toe of the dam. However, for the exceptional case where the dam is designed with a natural downstream blanket and with no downstream seepage control measures (relief wells or trench drains), upstream blankets should be designed so that the factor of safety against uplift or heaving at the downstream toe of the dam is at least 3. The rate of discharge through the pervious foundation per unit length of dam (see figure 9-19) is
where \( q_f \) is the rate of discharge through the pervious foundation per unit length of dam. The acceptable rate of discharge or underseepage depends upon the value of the water or hydropower lost, availability of downstream right-of-way, and facility for disposal of underseepage. The following procedure is used to determine the length of an upstream blanket when there is a downstream blanket present (see figure 9-19b):

(a) Determine \( L_1 \) from equation 9-14 using a conservative value of \( k_f/k_{BR} \), i.e., the highest probable ratio.

(b) Determine \( L_3 \) from equation 9-15 using a conservative value of \( k_f/k_{BL} \), i.e., the highest probable ratio.

(c) Determine \( h_o \), \( h_c \), and \( F_h \) from equations 9-16, 9-17, and 9-18, respectively. If \( F_h < 3.0 \), the blanket thickness of the upstream blanket may be increased, the permeability of the upstream blanket material may be decreased by compaction, or downstream seepage control measures may be used.

(d) Determine the rate of discharge through the pervious foundation per unit length of dam from equation 9-19. If the rate of discharge is excessive, a reduction can be obtained by increasing the thickness of the upstream blanket or reducing the permeability of the upstream blanket material by compaction. When these methods are used, steps 1 to 4 are repeated before going to step 5.

(e) If the rate of discharge is acceptable, calculate the factor

\[
c = \frac{1}{\sqrt{(k_f/k_{BR})bRd}}
\]

(9-20)

where \( c \) has the units of \( 1/ft \).

(f) Enter figure 9-20 with \( c \) and \( L_1 \) and obtain \( L_o \), which is the distance from the upstream toe of a homogeneous impervious dam or the impervious core section of a zoned embankment to where a discontinuity in the upstream blanket will have no effect on the uplift at the downstream toe of the dam or rate of discharge through the pervious foundation. This is the point beyond which a natural blanket may be removed in a borrowing operation. Also, \( L_o \) would represent the distance upstream from the toe of the dam to which a streambed should be blanketed to ensure the continuity of a natural upstream blanket. If there is no downstream blanket the pressure head under the blanket at the downstream toe of the dam will be zero (see equation 9-16).
and the following procedure is used to determine the length of the upstream blanket:

- Assume several values of \( L_0 \) (length of the upstream blanket from the upstream toe of a homogeneous impervious dam or the impervious core section of a zoned embankment).

Figure 9-20. Effective lengths of upstream and downstream impervious blankets (from U.S. Department of Agriculture).
• Calculate $c$ from equation 9-20 using the design thickness and permeability rates for the constructed blanket and pervious foundation. Note that $c$ has units of $1/\text{ft}$.

• Enter figure 9-20 with the assumed values of $L_o$ and the calculated values of $c$ to obtain the corresponding value of $L_1$ for each assumed value of $L_o$.

• Calculate $q_f$ from equation 9-19 ($L_3 = 0$ for no downstream blanket) using the values of $L_1$ obtained from figure 9-20.

• Plot $q_f$ versus $L_o$. The curve will indicate a rapid decrease in $q_f$ with increasing values up to a point where the curve flattens out indicating an optimum length. The upstream blanket can be terminated at any point where the desired reduction in rate of discharge through the pervious foundation per unit length of dam is achieved (Talbot and Nelson 1979).

c. Materials and Construction. At sites where a natural blanket of impervious soil already exists, the blanket should be closely examined for gaps such as outcrops of pervious strata, streambeds, root holes, boreholes, and similar seepage paths into the pervious foundation which, if present, should be filled or covered with impervious material to provide a continuous blanket to a distance $L_o$ from the upstream toe of the dam. Also, as previously stated, upstream borrow areas should be located greater than the distance $L_o$ from the upstream toe of the dam so as not to reduce the effectiveness of the natural blanket. Figure 9-21 shows the influence of gaps in the upstream blanket on relative seepage and uplift at the toe of the dam. That portion of the upstream blanket placed beneath the embankment to tie into the impervious core should be composed of the same material and compacted in the same manner as the core. Upstream of the embankment, the blanket is constructed by placing impervious soil in lifts and compacted only by movement of hauling and spreading equipment, or to whatever additional extent is necessary for equipment operation. Exposed clay blankets can shrink and crack after placement. If such cracks penetrate the blanket, they will reduce the effectiveness of the blanket. Thus it may become necessary to sprinkle the surface of the blanket to help retain moisture until a permanent pool is impounded. In higher reaches of abutments which are infrequently flooded by the reservoir, a thicker blanket may be required so that cracks will not fully penetrate the blanket. In colder climates, the blanket thickness should be increased to account for the loosening of the upper part of the blanket by frost action which substantially increases the permeability.

d. Reservoir Siltation. For some reservoirs, appreciable siltation occurs which may both increase the thickness of and lengthen the upstream blanket. Although the siltation may reduce the rate of discharge through the pervious foundation with time, it is not a factor to be counted upon in design because the upstream blanket must function adequately following initial filling of the reservoir prior to the occurrence of siltation.
9-6. Downstream Seepage Berms.

a. Introduction. When a complete cutoff is not required or is too costly, and it is not feasible to construct an upstream impervious blanket, a downstream seepage berm may be used to reduce uplift pressures in the pervious foundation underlying an impervious top stratum at the downstream toe of the dam. Other downstream underseepage control measures (relief wells or toe trench drains) are generally required for use with downstream seepage berms. Downstream seepage berms can be used to control underseepage efficiently where the downstream top stratum is relatively thin and uniform or where no top stratum is present, but they are not efficient where the top stratum is relative thick and high uplift pressures develop. Downstream seepage berms may vary in type from impervious to completely free draining. The selection of the type of downstream seepage berm to use is based upon the availability of borrow materials and relative cost of each type.

b. Design Considerations. When the top stratum or pervious foundation consists of several layers of different soils, they must be transformed into a single stratum with an effective thickness and permeability (see procedure given in U. S. Army Engineer Waterways Experiment Station 1956a). Where a
downstream natural blanket is present, the downstream seepage berm should have a thickness so that the factor of safety against uplift or heaving at the downstream toe of the dam is at least 3 and width so that the factor of safety against uplift at the downstream toe of the seepage berm is at least 1.5. Formulas for the design of downstream seepage berms where a downstream natural blanket is present are given in figure 9-22. If there is no downstream natural blanket present, the need for a downstream seepage berm will be based upon Bligh's creep ratio.

\[
c_B = \frac{X_1 + L_2 + X}{h}
\]  

(9-21)

where

\begin{align*}
  c_B & = \text{Bligh's creep ratio} \\
  X_1 & = \text{effective length of upstream blanket} \\
  L_2 & = \text{length of dam base} \\
  X & = \text{width of downstream seepage berm} \\
  h & = \text{net head on dam}
\end{align*}

Minimum acceptable values of Bligh's creep ratio are given in table 9-10. If the creep ratio is greater than the minimum value, a downstream seepage berm is not required. If the creep ratio is less than the minimum value, the width of the downstream seepage berm should be made such that the creep ratio is above the minimum value shown in table 9-10. The thickness of the downstream seepage berm at the toe of the dam will be determined so that the factor of safety against uplift or heaving at the downstream toe of the dam is at least 3. The pressure head beneath the downstream seepage berm at the landside toe of the levee is

\[
h_o = \frac{h(X + 0.43d)}{X_1 + L_2 + X + 0.43d}
\]  

(9-22)

where

\[
h_o = \text{pressure head under the seepage berm at the downstream toe of the dam}
\]

(1) A downstream seepage berm may be required to correct other problems such as excessive seepage gradients under the dam (could be detected by checking the rate of underseepage).
d = thickness of pervious foundation

\( X_1 = \text{effective length of upstream natural blanket (taken equal to 0.43d where no upstream natural blanket exists)} \)

The rate of discharge through the pervious foundation per unit length of dam is

\[
q_f = \frac{k_f \cdot d}{X_1 + L_2 + X + 0.43d}
\]  \( (9-23) \)

where

\( q_f = \text{rate of discharge through the pervious foundation per unit length of dam} \)

\( k_f = \text{horizontal permeability of pervious foundation} \)

As stated previously, the acceptable rate of discharge or underseepage depends upon the value of the water or hydropower lost, availability of downstream right-of-way, and facility for disposal of underseepage. Downstream seepage berms should have a minimum thickness of 10 ft at the dam toe and a minimum thickness of 5 ft at the berm toe. The computed thickness of the berm should be increased 25 percent to allow for shrinkage, foundation settlements, and variations in the design factors. Downstream seepage berms should have a slope of 1V on 50H or steeper to ensure drainage (U. S. Army Engineer Waterways Experiment Station 1956a).

c. Materials and Construction. As previously stated, the selection of the type of material used to construct the downstream seepage berm is based upon the availability of borrow materials and relative cost of each type. A berm constructed of impervious soil should be composed of the same material as the impervious core. That portion of the downstream impervious seepage berm placed beneath the embankment to tie into the impervious core should be compacted in the same manner as the core. Downstream of the embankment, the impervious seepage berm is constructed by placing impervious soil in lifts and compacting only by movement of hauling and spreading equipment, or to whatever additional extent is necessary for equipment operation. Semipervious material used to construct downstream seepage berms should have an in-place vertical permeability equal to or greater than that of the upstream natural blanket and are compacted in the same manner as described previously for impervious material. Material used in a sand berm should be as pervious as possible, with a minimum in-place vertical permeability of \( 100 \times 10^{-4} \) cm per sec. Downstream seepage berms constructed of sand should be compacted to an average in-place relative density of at least 85 percent with no portion of the berm having a relative density less than 80 percent. As proper functioning of a downstream seepage berm constructed of sand depends upon its continued perviousness, it should not be constructed until after the downstream slope of the earth dam has
Figure 9-22. Design of downstream seepage berms where a downstream natural blanket is present (from U.S. Army Engineer Waterways Experiment Station 120)
become covered with sod and stabilized so that soil particles carried by surface runoff and erosion will not clog the seepage berm. If it is necessary to construct the downstream seepage berm at the time the earth dam is built or before it has become covered with sod, an interceptor dike should be built at the intersection of the downstream toe of the dam and the seepage berm to prevent surface wash from clogging the seepage berm. A free-draining downstream seepage berm is one composed or random fill overlying horizontal sand and gravel drainage layers with a terminal perforated collector pipe system (U. S. Army Engineer Waterways Experiment Station 1956a).

9-7. Relief Wells.

a. Introduction. When a complete cutoff is not required or is too costly, relief wells installed along the downstream toe of the dam may be used to prevent excessive uplift pressures and piping through the foundation. Relief wells increase the quantity of underseepage from 20 to 40 percent depending upon the foundation conditions. Relief wells may be used in combination with other underseepage control measures (upstream impervious blanket or downstream seepage berm) to prevent excessive uplift pressures and piping through the foundation. Relief wells are applicable where the pervious foundation has a natural impervious cover. The well screen section (see figure 9-23), surrounded by a filter if necessary, should penetrate into the principal pervious stratum to obtain pressure relief, especially where the foundation is stratified. The wells, including screen and riser pipe, should have a diameter which will permit the maximum design flow without excessive head losses but in no instance should the inside diameter be less than 6 in. Filter fabrics should not be used in conjunction with relief wells (see Appendix D). Even in nearly homogeneous stratum, a penetration of less than 50 percent results in significant rise in pressure midway between adjacent wells, or requires close spacing. Relief wells should be located so that
Figure 9-23. Typical relief well (after EM 1110-2-1913)
their tops are accessible for cleaning, sounding for sand, and pumping to
determine discharge capacity. Relief wells should discharge into open ditches
or into collector systems outside of the dam base which are independent of toe
drains or surface drainage systems. Experience with relief wells indicates
that with the passage of time the discharge of the wells will gradually
decrease due to clogging of the well screen and/or reservoir siltation. A
comprehensive study of the efficiency of relief wells along the Mississippi
River levee showed that the specific yield of 24 test wells decreased 33 per-
cent over a 15-year period. Incrustation on well screens and in gravel filters
was believed to be the major cause (Montgomery 1972). Therefore, the amount of
well screen area should be designed oversized and a piezometer system installed
between the wells to measure the seepage pressure, and if necessary additional
relief wells should be installed (EM 1110-2-2300, U. S. Army Engineer Waterways
Experiment Station 1956a, Singh and Sharma 1976).

b. History of Use. The first use of relief wells to prevent excessive
uplift pressures at a dam was by the U. S. Army Engineer District, Omaha, when
21 wells were installed from July 1942 to September 1943 as remedial seepage
control at Fort Peck Dam, Montana. The foundation consisted of an impervious
stratum of clay overlying pervious sand and gravel. Although a steel sheetpile
cutoff was driven to shale, sufficient leakage occurred to develop high hydro-
static pressure at the downstream toe that produced a head of 45 ft above the
natural ground surface. This uplift pressure was first observed in piezometers
installed in the pervious foundation. The first surface evidence of the high
hydrostatic pressure came in the form of discharge from an old well casing that
had been left in place. Since it was important that the installation be made
as quickly as possible, 4- and 6-in. well casings, available at the site, were
slotted with a cutting torch and installed in the pervious stratum with solid
(riser) pipe extending to the surface. Wells were first spaced on 250-ft
centers and later intermediate wells were installed making the spacing 125 ft.
The hydrostatic pressure at the downstream toe was reduced from 45 to 5 ft and
the total flow from all wells averaged 10 cu ft per sec (U. S. Army Engineer
District, Omaha 1982). The first use of relief wells in the original design of
a dam was by the U. S. Army Engineer District, Vicksburg, when wells were
installed during construction of Arkabutla Dam, Mississippi, completed in June
1943. The foundation consisted of approximately 30 ft of impervious loess
underlain by a pervious stratum of sand and gravel. The relief wells were
installed to provide an added measure of safety with respect to uplift and
piping along the downstream toe of the embankment. The relief wells consisted
of 2-in. brass wellpoint screens 15 ft long attached to 2-in. galvanized
wrought iron riser pipes spaced at 25-ft intervals located along a line 100 ft
upstream of the downstream toe of the dam. The top of the well screens was
installed about 10 ft below the bottom of the impervious top stratum. The well
efficiency decreased over a 12-year period to about 25 percent primarily as a
result of clogging of the wells by influx of foundation materials into the
screens and/or the development of corrosion or incrustation. However, the
piezometric head along the downstream toe of the dam, including observations
made at a time when the spillway was in operation, has not been more than 1 ft
above the ground surface except at sta 190+00 where a maximum excess hydro-
static head of 9 ft was observed (U. S. Army Engineer Waterways Experiment
Station 1958). Since these early installations, relief wells have been used at
many dams to prevent excessive uplift pressures and piping through the foundation.

c. Design Considerations.

(1) General. The factors to be considered in determining the need for and designing a relief well system include characteristics of the landside top stratum; permeability, stratification, and depth of the pervious foundation in which seepage is to be controlled; net head acting on the dam; dimensions of the relief well system being considered; allowable factor of safety with respect to uplift at the downstream toe of the dam; and allowable rate of discharge through the pervious foundation. Some factors, like the net head acting on the dam, can be determined with good accuracy. Other factors like permeability and stratification are more difficult to assess. The design of the relief wells should be based on the best estimate of permeability values and then subjected to a sensitivity analysis using several values of permeability to ensure that the adopted design is adequate to intercept seepage and lower uplift pressures to the required extent allowing for the likelihood that the values of permeability used in design lake precision (Kaufman 1976). The area between the dam abutments is divided into reaches where geologic and soil conditions are assumed uniform within the reach (see figure 9-24). Generally, the design procedure for relief wells consists of determining the head which would exist along the downstream toe of the dam without relief wells, comparing this head to that desired for a given factor of safety, and designing a relief well system to reduce the head to the desired value. There is no unique solution because there is an infinite number of well systems (radius, penetration, spacing, etc.) which reduce the head to the given value. The objective is to select one which is economical, has reasonable dimensions, and can produce the desired results. Usually the designer selects the radius and penetration and then determine the required spacing of the well system. This becomes an iterative procedure wherein the designer assumes a value of well spacing, computes the head between wells and repeats this for several trial spacings until a spacing is found that produces the desired head along the downstream toe of the dam. The cost of the well system is determined and then a design can be prepared for a different penetration to determine if some economy can be achieved by changing the penetration of the system. Fully penetrating relief wells are often used in aquifers up to about 75 ft thick. For larger depths of pervious strata, it is usually more economical to have well systems with 50 percent or greater penetration at closer spacing. The equations for relief well design depend upon the values of the source of seepage and seepage exit length as shown in figure 9-25. The source of seepage is assumed to be a line source parallel to the well system and the dam axis. The location of the source of seepage depends upon the thickness and vertical permeability of any natural top stratum upstream of the dam and any impervious blanket constructed upstream of the dam, the permeability and thickness of the pervious foundation, and the presence of any borrow pits and/or major erosion features which reduce the thickness of the top stratum (see procedure to evaluate the source of seepage given in U. S. Army Engineer Waterways Experiment Station 1956a). The value of the seepage exit \( X_3 \) in figure 9-25 depends upon the thickness and permeability of the top stratum downstream from the toe of the dam, the thickness and permeability of the pervious substratum,
and the presence of any geologic features and/or man-made features which would result in an open or blocked seepage exit. The procedure for computation of the seepage exit distance, rate of discharge through the pervious foundation per unit length of dam, and pressure head without relief wells is given in figure 9-26. Generally relief wells have diameters of 6 to 18 in. and screen lengths of 20 to 100 ft, depending on the requirements. Some types of screens used for wells are slotted or perforated steel pipe, perforated steel pipe wrapped with steel wire, slotted wood stave pipe, and slotted plastic pipe. Riser pipe usually consist of the same material as the screen but does not contain slots or perforations. The open area of a well screen should be sufficiently large to maintain a low entrance velocity (< 0.1 ft per sec) at the design flow in order to minimize head losses across the screen and reduce the incrustation and corrosion rates. The entrance velocity is calculated by dividing the expected or desired yield of the relief well by the area of openings in the screen (Johnson Division, Universal Oil Products Co. 1972). Filter packs around relief wells are usually 6 to 8 in. thick and must meet the criteria specified in Appendix D. Head losses within the relief well system consist of entrance head loss, friction head loss in the screen and riser pipes, and velocity head loss as shown in figure 9-27. The effective well radius is that radius which would exist if there were no hydraulic head loss into the well. For a well without a filter, the effective well radius is one-half the outside diameter of the well screen. Where a filter has been placed around the well, the effective well radius is the outside radius of the well screen plus one-half of the thickness of the filter.
(2) Effective Well Penetration. In a stratified foundation, the effective well penetration usually differs from that computed from the ratio of the length of well screen to the total thickness of the aquifer. The procedure for determining the required length of well screen to achieve an effective penetration in a stratified aquifer is as follows. Each stratum of the previous foundation is first transformed into an isotropic layer (Leonards 1962)

\[ d = d' \sqrt{\frac{k_H}{k_V}} \]  

(9-24)

where

- \( d \) = transformed layer thickness
- \( d' \) = actual layer thickness
- \( k_H \) = horizontal permeability of layer
- \( k_V \) = vertical permeability of layer
Figure 9-26. Computation of rate of discharge and pressure heads for semi-pervious downstream top stratum and no relief wells (from U. S. Army Engineer Waterways Experiment Station).
TOTAL HYDRAULIC HEAD LOSS IN A WELL \( (H_w) \) IS

\[
H_w = H_s + H_f + H_r + H_v
\]

WHERE

- \( H_s \) = ENTRANCE HEAD LOSS (SCREEN AND FILTER) ESTIMATE FROM CURVE a.
- \( H_f \) = HEAD LOSS IN SCREENED SECTION OF WELL ESTIMATE FROM CURVE b FOR A DISTANCE OF ONE-HALF THE SCREEN LENGTH.
- \( H_r \) = HEAD LOSS WITHIN THE RISER AND CONNECTIONS. ESTIMATE FROM CURVE b.
- \( H_v \) = VELOCITY HEAD LOSS. ESTIMATE FROM CURVE c.

The value of \( H_v \) must be subtracted from the computed value of \( H_w \) to obtain the lift or water level in a well.

Figure 9-27. Hydraulic head loss in relief wells (after Leonards 205)
The transformed permeability of each layer of the pervious foundation is

\[ \bar{k} = \sqrt{k_H k_V} \]  

(9-25)

where \( \bar{k} \) is the transformed permeability of layer. The thickness of the transformed, homogeneous, isotropic pervious foundation is

\[ D = \sqrt{\sum (d'k_H) / \sum (d'/k_V)} \]  

(9-26)

where \( D \) is the thickness of pervious foundation. The effective permeability of the transformed pervious foundation is

\[ k = \frac{\sqrt{\sum (d'k_H)}}{\sum (d'/k_V)} \]  

(9-27)

where \( k \) is the effective permeability of transformed pervious foundation. The effective well screen penetration into the transformed pervious foundation is

\[ w = \frac{\sum d'k_H}{k} \]  

(9-28)

where

\[ w = \text{effective well screen penetration into transformed pervious foundation} \]

\[ \bar{w} = \text{actual well screen length} \]

The percent penetration of the well screen into the transformed pervious foundation is

9-70
where $\bar{D}$ is the actual pervious foundation thickness.

(3) Factor of Safety. The factor of safety against uplift or heaving at the downstream toe of the dam, based upon the critical gradient, is

$$\frac{W}{D}, \ z = \frac{100 \sum_{o}^{100} d'k'_{H}}{kd} = \frac{100 \sum_{o}^{100} d'k'_{H}}{\sum_{o}^{D}}$$  \hspace{1cm} (9-29)

where

- $F_h$ = factor of safety against uplift or heaving at the downstream toe of the dam
- $i_{CR}$ = critical upward hydraulic gradient under the top stratum at the downstream toe of the dam
- $i_o$ = allowable upward hydraulic gradient under the top stratum at the downstream toe of the dam
- $\gamma_{sub}$ = submerged unit weight of downstream top stratum soil
- $h_a$ = allowable pressure head under the top stratum at the downstream toe of the dam
- $Z_{bL}$ = thickness of downstream top stratum
- $\gamma_w$ = unit weight of water

The factor of safety against uplift or heaving at the downstream toe of the dam provided by the relief well system should be at least 1.5\(^{(1)}\).

(4) Infinite and Finite Relief Well Systems. Formulas for the design of relief wells are based on the assumption that the flow is laminar,

\(^{(1)}\) Relief wells should be designed to reduce the excess head to zero to prevent upward seepage from occurring beneath the downstream top stratum.
artesian, and continuous and that a steady-state condition exists. Relief well systems are considered to be infinite or finite in length. The term infinite is applied to a system of wells that conforms approximately to the following idealized conditions:

(a) The wells are equally spaced and identical in dimensions.

(b) The pervious foundation is of uniform depth and permeability along the entire length of the system.

(c) The effective source of seepage and the effective line of downstream exit are parallel to the line of wells.

(d) The boundaries at the ends of the relief wells are impervious, normal to the line of wells, and at a distance equal to one-half the well spacing beyond the end wells of the system.

If these conditions exist, the flow to each well and the pressure distribution around each well are uniform for all wells along the line. Therefore, there is no flow across planes centered between wells and normal to the line, hence no overall longitudinal component of flow exists anywhere in the system. The term infinite is applied to such a system because it may be analyzed mathematically by considering an infinite number of wells; the actual number of wells in the system may be from one to infinity. Normally, a line of relief wells below a dam extending entirely across a valley and terminating at relatively impervious valley walls should be designed as an infinite line. A finite system of wells in any system that does not approximate the idealized condition for the infinite system. Whenever a major and abrupt change in the character of the system such as penetration or well spacing might result in an appreciable component of flow parallel to the line of wells, the use of design procedures for finite systems will be used (see U. S. Army Corps of Engineers 1963).

(5) Drawdown to Infinite Line of Fully Penetrating Relief Wells with Impervious Top Stratum. Where the flow to an infinite line of fully penetrating relief wells is from an infinite line source and the top stratum is assumed to be completely impervious, \(^{(1)}\) as shown in figure 9-28. The drawdown produced by an equivalent continuous slot is

\[
H - h_e = \frac{Q L}{k D a}
\]  

\[(9-31)\]

\(^{(1)}\) Also applicable—when the top stratum is semipervious provided the well system is located in a drainage ditch and the head is kept below the ground surface on the downstream side of the dam to prevent any seepage upward through the top stratum. Under these conditions, the downstream top stratum acts as if it is impervious.
where

\[ H - h_e = \text{drawdown produced by flow from continuous slot} \]
\[ Q_w = \text{discharge from equivalent continuous slot} \]
\[ L = \text{distance from line source of seepage to wells} \]
\[ k = \text{effective permeability of transformed pervious foundation} \]
\[ D = \text{thickness of transformed pervious foundation} \]
\[ a = \text{well spacing} \]

However, an additional head occurs because of converging flow at the wells. This head loss is a function of well flow, well spacing and penetration, well radius, and thickness and permeability of the pervious foundation. For fully penetrating wells

\[ \Delta h_w = \frac{Q_w}{2\pi k D} \ln \frac{a}{2\pi r_w} \quad (9-32) \]

where

\[ \Delta h_w = \text{head loss at well due to converging flow (see figure 9-28)} \]
\[ r_w = \text{effective radius of well (outside radius of well screen plus one-half of the thickness of the filter)} \]

The total drawdown at the well, neglecting hydraulic head losses in the well, is that at the slot plus that due to the well

\[ H - h_w = H - h_e + \Delta h_w \quad (9-33) \]

Substituting equations 9-31 and 9-32 into equation 9-33

\[ H - h_e = \frac{Q_w L}{k D a} + \frac{Q_w}{2\pi k D} \ln \frac{a}{2\pi r_w} \quad (9-34) \]
Figure 9-28. Flow to an infinite line of fully penetrating relief wells from an infinite line source of seepage (after Leonards\textsuperscript{205})

The head midway between wells will exceed the head at the well by

\[
\Delta h_m = \frac{Q_w}{2\pi kD} \ln \frac{a}{2\pi r_w} \tag{9-35}
\]

where \(\Delta h_m\) is the head increase midway between wells. The drawdown midway between wells is

\[
H - h_m = \frac{Q_w}{kD} - 0.11 \frac{Q_w}{kD} \tag{9-36}
\]

At a distance downstream from the well system, the head will exceed that at the well by
where $\Delta h_D$ is the head increase downstream of wells. The drawdown downstream of the wells is

$$\Delta h_D = \frac{Q_w}{2\pi k D} \ln \frac{a}{2\pi r_w}$$

(9-37)

$$H - h_D = \frac{h_D - h_w}{\frac{a}{2\pi L} \ln \frac{a}{2\pi r_w}}$$

(9-38)

(6) Drawdown to Infinite Line of Partially Penetrating Relief Wells with Impervious Top Stratum. For an infinite line of partially penetrating relief wells where the flow is from an infinite line source and the top stratum is assumed to be impervious (or semipervious as previously described) the head loss at the partially penetrating well due to converging flow is

$$\Delta h_w = \frac{Q_w \theta_{avg}}{kD}$$

(9-39)

where $\theta_{avg}$ is the average uplift factor (obtained from figure 9-29). The total drawdown at the partially penetrating well, neglecting hydraulic head losses in the well, is that at the slot plus that due to the well

$$H - h_w = \frac{Q_w}{kD} \left( \frac{L}{a} + \theta_{avg} \right)$$

(9-40)

The head midway between partially penetrating wells will exceed the head at the well by

$$\Delta h_m = \frac{Q_w}{kD} \frac{\theta_m}{a}$$

(9-41)

where

$\theta_m$ is the midpoint uplift factor (obtained from figure 9-29). The drawdown midway between partially penetrating wells is

$$H - h_m = \frac{Q_w}{kD} \left( \frac{L}{a} + \theta_{avg} - \theta_m \right)$$

(9-42)
Figure 9-29. Nomograph for obtaining uplift factors for design of partially penetrating relief wells (from U.S. Army Engineer Waterways Experiment Station)
(7) Drawdown to Infinite Line of Relief Wells with Semipervious Top Stratum. Where the top stratum is semipervious, the need for relief wells is evaluated by determining the piezometric grade line without relief wells using blanket formulas given in figure 9-26. As stated previously, the factor of safety against uplift or heaving at the downstream toe of the dam, as determined from equation 9-30, should be at least 1.5. If relief wells are required, the spacing for an infinite line of relief wells for a given penetration is determined using a procedure of successive trials and the nomograph given in figure 9-29. The required well spacing is affected by hydraulic head losses in the well which are estimated from figure 9-27. The procedure for computing the well spacing is as follows:

(a) Compute the allowable pressure head under the top stratum at the downstream toe of the dam from

\[ h_a = \frac{\gamma_{sub} Z_{bl}}{\gamma_w F_h} \]  \hspace{1cm} (9-43)

where

- \( h_a \) = allowable pressure head under the top stratum at the downstream toe of the dam
- \( \gamma_{sub} \) = submerged unit weight of downstream top stratum soil
- \( Z_{bl} \) = thickness of downstream top stratum
- \( \gamma_w \) = unit weight of water
- \( F_h \) = factor of safety against uplift or heaving at the downstream toe of the dam

(b) Assume that the net head in the plane of the wells equals the allowable pressure head under the top stratum at the downstream toe of the dam and compute the net seepage gradient toward the well line

\[ \Delta M = \frac{h - H_{avg}}{S} - \frac{H_{avg}}{X_3} \] \hspace{1cm} (9-44)

where

- \( \Delta M \) = net seepage gradient toward the well line
- \( h \) = net head acting on the dam
- \( H_{avg} \) = net head in the plane of the wells
S = distance from line of relief wells to effective source of seepage entry (see procedure in U. S. Army Engineer Waterways Experiment Station 1956a)

\(X_3\) = distance from line of relief wells to effective seepage exit (see procedure in figure 9-26)

Setting \(H_{avg} = h_a\) in equation 9-44 gives

\[
\Delta M = \frac{h - h_a}{S} - \frac{h_a}{X_3}
\]  
(9-45)

(c) Assume a well spacing and compute the flow from a single well

\[
Q_w = \frac{h k_f D}{S + \left(\frac{X_3}{S + X_3}\right) \theta_{avg}} = a \Delta M k_f D \]
(9-46)

where

\(Q_w\) = flow from a single well
\(k_f\) = effective permeability of transformed pervious foundation
\(D\) = transformed thickness of pervious foundation
\(a\) = well spacing
\(\theta_{avg}\) = average uplift factor (obtained from figure 9-29)

(d) Estimate the total hydraulic head loss in the well from figure 9-27.

(e) Compute the net average head in the plane of wells above the total head loss in the well including elevation head loss (see figure 9-25) from

\[
h_{avg} = H_{avg} - H_w
\]
(9-47)

where

\(h_{avg}\) = net average head in the plane of wells above the total head loss in the well including elevation head loss
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\[ H_{avg} = \text{net head in the plane of wells} \]

\[ H_w = \text{total head loss in the well including elevation head loss} \]

(f) Substitute values obtained from \( \Delta M \) and \( h_{avg} \) from equation 9-45 and 9-47, respectively, and solve for the average uplift factor

\[
\theta_{avg} = \frac{h_{avg}}{a \Delta M}
\]

(9-48)

where \( \theta_{avg} \) is the average uplift factor.

(g) Find \( \theta_{avg} \) from figure 9-29 using the values of \( a \) used in equation 9-48 and the corresponding \( a/r_w \) and \( D/a \) values.

(h) The first trial well spacing is that of value \( a \) for which \( \theta_{avg} \) from step (f) equals \( \theta_{avg} \) from step (g).

(i) Find \( \theta_m \) from figure 9-29 for the first trial well spacing and the corresponding values of \( a/r_w \) and \( D/a \).

(j) If \( \theta_{avg} > \theta_m \), repeat steps (c) to (i) using the first trial well spacing in lieu of the spacing originally used in step (c), and determine the second trial well spacing. This procedure should be repeated until relatively consistent values of \( a \) are obtained on two successive trials. Usually the second trial spacing is sufficiently accurate.

If in step (j), \( \theta_{avg} > \theta_m \), a modified procedure is used for the second trial:

(k) Assume \( H_m = h_a \) and compute \( Q_w \) from equation 9-46 using the value of \( \Delta M \) obtained in step (b) and the first trial well spacing from step (h).

(l) Estimate \( H_w \) from \( Q_w \) of step (k) and figure 9-27.

(m) Compute the net head beneath the top stratum midway between the wells above the total head loss in the well including elevation head loss (see figure 9-25) from

\[
h_m = H_m - H_w
\]

(9-49)

where

\( h_m = \text{net head beneath the top stratum midway between the well above the total head loss in the well including elevation head loss} \)
HM = net head beneath the top stratum midway between the wells
HW = total head loss in the well including elevation head loss

(n) Using $\theta_{avg}$ and $\theta_m$ from steps (h) and (i), respectively, compute $h_{avg}$ from

$$h_{avg} = \frac{\theta_{avg}}{\theta_m}$$

(9-50)

where $h_{avg}$ is the net head in the plane of wells.

(o) Using $H_w$ and $h_{avg}$ from steps (1) and (n), respectively, and compute $H_{avg}$ from

$$H_{avg} = H_w + h_{avg}$$

(9-51)

(p) Compute $\Delta M$ from equation 9-44 using $H_{avg}$ from step (o).

(q) Using $h_m$ and $\Delta M$ from steps (m) and (p), respectively, compute $\theta_m$ for various values of a from

$$\theta_m = \frac{h_m}{a\Delta M}$$

(9-52)

where $\theta_m$ is the midpoint uplift factor.

(r) Find $\theta_m$ from figure 9-29 for the values of a used in step (q) and the corresponding $a/r_w$ and $D/a$ values.

(s) The second trial well spacing is that value of a which $\theta_m$ from step (q) equals $\theta_m$ from step (r).

(t) Find $\theta_{avg}$ from figure 9-29 for the second trial well spacing and the corresponding values of $a/r_w$ and $D/a$.

(u) Determine the third trial well spacing by repeating steps (k) to (t) using the second trial well spacing in lieu of the spacing originally assumed in step (k), and in step (n) using the values of $\theta_m$ and $\theta_{avg}$ from steps (s) and (t), respectively, instead of those from steps (h) and (i). This procedure should be repeated until relatively consistent values of a are obtained on two successive trials. Normally, the third trial is sufficiently accurate.
(8) Drawdown to Finite Line of Relief Wells. In a short, finite line of relief wells, the heads midway between wells exceed those for an infinite line of wells both at the center and near the ends of the well system as shown in figure 9-30. Note that the pressures between wells, or midpoint pressures, are lower at the center of the well system and gradually increase towards the end of the line. With an infinite line of wells, the heads midway between wells are constant along the entire length of the well line. Numerous well systems may be fairly short, and for these it will be necessary to reduce the well spacing computed for an infinite line of wells so that heads midway between wells will not be more than the allowable pressure head under the top stratum at the downstream toe of the dam. The ratio of the head midway wells at the center of finite well systems to the head between wells in an infinite line of wells, for various well spacings and seepage exit lengths, is given in figure 9-31. The spacing of relief wells in a finite line should be the same as that required in an infinite line of wells to reduce the head midway between wells to $h_a$ divided by the ratio of $H_{m_n}/H_{m_\infty}$ from figure 9-31. In any finite line of wells of constant penetration and spacing, the head midway between wells near the ends of the system exceeds that at the center of the system. Thus at the ends of both short and long well systems, the relief wells should generally be made deeper to provide additional penetration of the pervious substratum so as to obtain the same head reduction as in the central part of the well line. The above-mentioned procedures for designing finite relief well systems, although approximate, are usually sufficient. More exact, but more complex, procedures are available (see U. S. Army Corps of Engineers 1963).

Figure 9-30. Variation of pressure relief along a finite line of relief wells (after EM 1110-2-1905)
Assumptions:

1. Penetration of pervious aquifer by well screen = 100%
2. Semipervious top stratum infinite in landward extent
3. Effective well radius = 1.0 ft

Note:

1. \( H_m \) = Head midway between wells in infinite well line
2. \( H_n \) = Head midway between wells at center of a finite well line
3. \( a \) = Well spacing in feet
4. \( x_3 \) = Exit length in feet

Figure 9-31. Ratio of head midway between relief wells at center of a finite well system to head midway between wells in an infinite system (from U.S. Army Engineer Waterways Experiment Station 120)
d. Installation. While the specific materials used in the construction of relief wells and methods of installation differ, relief wells are basically very similar. They consist of a boring to facilitate the installation, a screen or slotted pipe section to allow the entrance of ground water, a filter to prevent entrance and ultimate loss of foundation material, a riser pipe to conduct the water to the ground surface, a check valve to prevent backflooding and entrance of foreign material detrimental to the installation, backfill to prevent recharge of the formation by surface water, a bottom plug to prevent inflow of soil, a V-notch weir at the top of the relief well to facilitate measurement of flow, and a cover and some type of barricade protection to prevent vandalism and damage to the top of the well by maintenance crews, livestock, etc. (see figure 9-23). Following development of the relief well, a pumping test should be conducted to determine the specific yield of the well and the amount of sand infiltration. Information from the pumping test is used to determine the acceptability of the well and for evaluating any changes in performance or loss of efficiency with time. Procedures for installation, development, and pumping tests are given in EM 1110-2-1913. A guide specification for relief wells is available.

e. Monitoring. As mentioned previously, the discharge of relief wells gradually decreases with time due to clogging of the well screen and/or reservoir siltation. Piezometers should be installed between relief wells to determine the seepage pressure in the main pervious strata. Relief wells should be sounded for sand and pumped to determine their discharge capacity under varying reservoir levels (see Chapter 13). A trend toward fall in relief well discharge accompanied by a fall in piezometric levels indicates a decrease in underseepage due to reservoir siltation and is favorable. However, a decrease in relief well discharge accompanied by a rise in piezometric levels indicates clogging of the relief wells and immediate rehabilitation and/or replacement of the wells or installation of additional wells is required (Singh and Sharma 1976). The operation, maintenance, and rehabilitation of relief wells is discussed in Chapter 14.

9-8. Trench Drain.

a. Introduction. When a complete cutoff is not required or is too costly, a trench drain may be used in conjunction with other underseepage control measures (upstream impervious blanket and/or relief wells) to control underseepage. A trench drain is a trench generally containing a perforated collector pipe and backfilled with filter material (see figure 9-32). Trench drains are applicable where the top stratum is thin and the pervious foundation is shallow so that the trench can penetrate into the aquifer. The existence of moderately impervious strata or even stratified fine sands between the bottom of the trench drain and the underlying main sand aquifer will render the trench drain ineffective. Where the pervious foundation is deep, a trench drain of practical depth would only attract a small portion of underseepage, and detrimental underseepage would bypass the drain and emerge downstream of the drain, thereby defeating its purpose. Trench drains may be used in

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(1) For partially penetrating relief wells, the bottom plug should be such that future screen extension will be possible,
conjunction with relief well systems to collect seepage in the upper pervious foundation that the deeper relief wells do not drain. If the volume of seepage is sufficiently large, the trench drain is provided with a perforated pipe. A trench drain with a collector pipe also provides a means of measuring seepage quantities and of detecting the location of any excessive seepage (U. S. Army Engineer Waterways Experiment Station 1956a, EM 1110-2-1911, EM 1110-2-1913, and Cedergren 1977).

b. **Location and Geometry.** Trench drains are generally located at the downstream toe of the dam as shown in figures 9-32a and 9-32c, but are sometimes located beneath the downstream slope of the dam as shown in figure 9-32b. Trench geometry will depend on the volume of expected underseepage, desired reduction in uplift pressure, construction practicalities, and the stability of the material in which the trench is to be excavated. Trenches with widths as small as 2 to 6 ft have been used. However, narrow trench widths require special compaction equipment (EM 1110-2-1913).

c. **Design Considerations.** The maximum head at the base of an impermeable top stratum downstream of a toe trench drain overlaying a homogeneous, isotropic, pervious foundation may be computed from figure 9-33. The distance to the source of seepage may be evaluated using the procedure given in U. S. Army Engineer Waterways Experiment Station 1956a. If the pervious foundation is stratified, it is transformed into an isotropic layer, as described previously (see Equations 9-24 to 9-27) prior to using figure 9-33. The factor of safety against uplift or heaving at downstream toe of the dam provided by the trench drain should be at least 1.5. If the downstream top stratum is semi-pervious, seepage into the trench, and the maximum head landward of the trench, will be somewhat less than that computed from figure 9-33 giving a slightly conservative design. When there is no downstream top stratum, seepage flow into the trench can be estimated from a flow net analysis (U. S. Army Engineer Waterways Experiment Station 1956a).

d. **Construction.** A trench drain usually contains a perforated pipe, surrounded by filter gravel, and backfilled with sand as shown in figure 9-34. Materials in trench drain must satisfy the filter gradation criteria given in Appendix D. As filter materials are placed, they must be protected from contamination resulting from inwash that might occur during a rainfall. The same control procedures are used for trench drains as those used in construction of pervious fill in the main embankment (EM 1110-2-1911).

9-9. **Concrete Galleries.** Internal reinforced concrete galleries have been used in earth and rockfill dams built in Europe, for grouting drainage, and monitoring of behavior. Galleries have not been constructed in embankment dams built by the Corps of Engineers to date. Some possible benefits to be obtained from the use of galleries in earth and rockfill dams are as follows (Sherard et al. 1963):

a. Construction of the embankment can be carried out independently of the grouting schedule.
Figure 9-32. Trench drains to control underseepage (from EM 1110-2-1913)
Figure 9-33. Design of toe trench drains for homogeneous, isotropic, pervious foundation, and for an impervious downstream top stratum
(from U. S. Army Engineer Waterways Experiment Station)

b. Drain holes drilled in the rock foundation downstream from the grout curtain can be discharged into the gallery and observations of the quantities of seepage in these drain holes will indicate where foundation leaks are occurring.

c. Galleries provide access to the foundation during and after reservoir filling so that additional grouting or drainage can be installed, if required, and the results evaluated from direct observations.

d. The additional weight of the overlying embankment allows higher grout pressures to be used.

e. Galleries can be used to house embankment and foundation instrumentation outlets in a more convenient fashion than running them to the downstream toe of the dam.

f. If the gallery is constructed in the form of a tunnel below the rock surface along the longitudinal axis of the dam, it serves as an exploratory tunnel for the rock foundation.
The minimum size cross section recommended for galleries and access shafts is 8 by 8 ft to accommodate drilling and grouting equipment. A gutter located along the upstream wall of the gallery along the line of grout holes will carry away cuttings from the drilling operation and waste grout from the grouting operation. A gutter and system of weirs located along the downstream wall of the gallery will allow for determination of separate flow rates for foundation drains (EM 1110-2-3502, and Blind 1982).
CHAPTER 10
SEEPAGE CONTROL THROUGH EARTH ABUTMENTS ADJACENT TO STRUCTURES
AND BENEATH SPILLWAYS AND STILLING BASINS

10-1. Through Earth Abutments. Earth and rock-fill dams, particularly in
 glaciated regions, may have pervious material, resulting from filling of the
 preglacial valley with alluvial or morainal deposits followed by the down-
cutting of the stream, in one or both abutments (Twelker 1957). Seepage
 through the pervious abutment(s) is combined with through seepage and under-
seepage to determine the total seepage loss. As mentioned previously, the pur-
pose of the project, i.e., long-term storage, flood control, hydropower, etc.,
will determine the allowable seepage loss. Seepage control through earth
abutments is provided by extending the upstream impervious blanket in the
lateral direction to wrap around the abutment up to the maximum water surface
elevation, by placing a filter layer between the pervious abutment and the dam
downstream of the impervious core section, and, if necessary, by installing
relief wells at the downstream toe of the pervious abutment. At the North
Branch of Kokosing Dam, Ohio, the left abutment is an outwash terrace consist-
ing of sands and gravels with layers of silt and clay as shown in figure 10-1.
Seepage control through the pervious abutment was provided by a 5-ft-thick
impervious upstream blanket which wrapped around the left abutment, a filter
layer between the pervious abutment and the dam downstream of the impervious
core (see figure 10-2), and three fully penetrating relief wells at the down-
stream toe of the pervious abutment (U. S. Army Engineer District, Huntington
1969).

10-2. Adjacent to Outlet Conduits. When the dam foundation consists of com-
pressible soils, the outlet works tower and conduit should be founded upon or
in stronger abutment soils or rock. When conduits are laid in excavated
trenches in soil foundations, concrete seepage cutoff collars shall not be
provided solely for the purpose of increasing seepage resistance since their
presence often results in poorly compacted backfill around the conduit. Col-
lars, with a minimum projection from the conduit surface, will be used over
conduit joints to protect against joint displacements resulting from differen-
tial movement on yielding foundations. Excavations for outlet conduits in
soil foundations shall be wide enough to allow for backfill compaction
parallel to the conduit using heavy rolling compaction equipment. Equipment
used to compact along the conduit should be free of framing that prevents its
load transferring wheels or drum from working against the structure. Excavated
slopes in soil for conduits should be no steeper than 1V to 2H to facilitate
adequate compaction and bonding of backfill with the sides of the excavation.
Drainage layers should be provided around the conduit in the downstream zone of
embankments without pervious shells. A concrete plug shall be used as backfill
in rock cuts for cut-and-cover conduits within the core zone to ensure a water-
tight bond between the conduit and vertical rock surfaces. The plug, which can
be constructed of lean concrete, should be at least 50 ft long and extend up to
the original rock surface. In embankments having a random or an impervious
downstream shell, horizontal drainage layers should be placed along the sides
and over the top of conduits downstream of the impervious core. Where outlet
structures are to be located in active seismic areas, special attention must be
given to the possibility of movement along-existing or possibly new faults
(EM 1110-2-2300).
Figure 10-1. Profile of left pervious abutment at North Branch of Kokosing Dam, Ohio (from U. S. Army Engineer District, Huntington).
10-3. Beneath Spillways and Stilling Basins. Adequate drainage should be provided under floor slabs for spillways and stilling basins to reduce uplift pressures. For soil foundations, a drainage blanket under the slab with transverse perforated pipe drains discharging through the walls or floor is generally provided, supplemented in the case of stratified foundations by deep well systems. Usually drainage of a slab on rock is accomplished by drain holes drilled in the rock with formed holes or pipes through the slab. The drainage blanket is designed to convey the seepage quickly and effectively to the transverse collector drains. It is designed as a graded reverse filter with coarse stones adjacent to the perforated drain pipe and finer material adjacent to the concrete structure to prevent the migration of fines into the drains. Outlets for transverse drains in the spillway chute discharge through the walls or floor at as low an elevation as practical to obtain maximum pressure reduction. Wall outlets should be 1 ft minimum above the floor to prevent blocking by debris. Cutoffs are provided at each transverse collector.
pipe to minimize buildup of head in case of malfunction of the pipe drain. Drains should be at least 6 in. in diameter and have at least two outlets to minimize the chance of plugging. Outlets should be provided with flat-type check valves to prevent surging and the entrance of foreign matter in the drainage system. For the stilling basin floor slab, it may be advantageous to place a connecting header along each wall and discharge all slab drainage into the stilling basin just upstream from the hydraulic jump at the lowest practical elevation, in order to secure the maximum reduction of uplift for the downstream portion of the slab. A closer spacing of drains is usually required than in the spillway chute because of greater head and considerable difference in water depth in a short distance through the hydraulic jump. Piezometers should be installed in the drainage blanket and deeper strata, if necessary, to monitor the performance of the drainage systems. If the drains or wells become plugged or otherwise noneffective, uplift pressures will increase which could adversely affect the stability of the structure (EM 1110-2-2400 and EM 1110-2-2300).
CHAPTER 11
SEEPAGE CONTROL IN ROCK FOUNDATIONS AND ABUTMENTS

11-1. General Considerations.

a. The choice of seepage control methods to use in rock foundations and abutments is dependent on a number of factors. Characterization of the foundation or abutment and identification of potential seepage paths is essential. Before any method of seepage control is implemented, the area must be thoroughly explored and tested to assure that the method chosen will apply to the general conditions as well as the conditions locally encountered and will serve the intended purpose. In many cases, a combination of methods can be used to the best advantage for rock foundations or abutments. The use of different control methods becomes particularly important when there is a change in the character of the foundation from one location to another, or a change in seepage characteristics between the foundation and the abutment.

b. Seepage should be cut off or controlled by drainage whenever economically possible. Safety, however, must be the governing factor for selecting a seepage control method. It should be noted that the possibility exists for control measures to cause substantial increases in seepage rather than decreases. Such increases are normally accompanied by reductions in uplift pressures and are therefore desirable if the increased seepage produces no detrimental side effects. In the final choice of a seepage control method, or methods, economic factors must be recognized and evaluated.

11-2. Cutoff Trenches.

a. No cutoff is 100 percent impervious and therefore the reduction in seepage from cutoff trenches is a relative matter. Cutoff trenches are normally employed where the character of the foundation is such that the construction of a satisfactory or effective grout curtain is not practical. Such trenches, when constructed, are normally backfilled with compacted impervious material, bentonite slurry, or neat cement.

b. Construction of trenches in rock foundations and abutments normally involves blasting using the presplit method with primary holes deck-loaded according to actual foundation conditions. After blasting, excavation is normally accomplished with a backhoe. Cutoff of seepage within the foundation is obtained by connecting an impervious portion of the foundation to the impervious portion of the structure by backfilling the trench with an impervious material. In rock foundations, as in earth foundations, the impervious layer of the foundation, in some cases, may be sandwiched between an upper and a lower pervious layer, and a cutoff to such as impervious layer would reduce seepage only through the upper pervious layer. However, where the thicknesses of the impervious and upper pervious are sufficient, the layers may be able to resist the upward seepage pressures existing in the lower pervious layer and thus remain stable. Cutoff of seepage within the abutment is normally obtained by extending the cutoff from above the projected seepage line to an impervious layer within the abutment. The type of backfill material is normally dictated by condition of the foundation or abutment, economics, and degree of cutoff required.
11-3. Abutment Impervious Blankets.

a. Impervious blankets overlying the upstream or riverside face of pervious abutments, or foundations, are effective in reducing the quantity of seepage and to some extent will reduce uplift pressures and gradients downstream. An impervious blanket may be used for earthen or rock abutments; however, a filter material is normally required with rock abutments.

b. The construction of impervious blankets is particularly adaptable to treating exposed pervious areas of abutments which are adjacent to the main structure. In cases where a natural impervious blanket exists on the abutment, ranging in depth from a few feet to many feet, full advantage should be taken of the existing material. Upstream borrow along the abutment should be controlled to prevent excessive excavation of the natural impervious top blanket. Conversely, localized areas which are thin and weak should be reinforced by the addition of additional impervious material.

c. Blankets may sometimes give adequate control of seepage water for low head structures, but for high head structures it is usually necessary to incorporate a downstream drainage system as a part of the overall seepage-control design. The benefits derived from abutment impervious blankets are due to the dissipation of a part of the reservoir head through the blanket. The proportion of head dissipated is dependent upon the thickness, length, and effective permeability of the blanket in relation to the permeability of the adjacent soil, or rock.

11-4. Drainage and Grouting Galleries and Tunnels.

a. Foundation galleries and tunnels in concrete gravity dams provide an exit for foundation drains and convenient facilities for rehabilitation work, or supplemental grouting, if required. The depth of drainage galleries, or exit elevation of the drains, with respect to the tailwater, controls the uplift downstream of the drains or wells. Generally, the lower the elevation of the gallery, the more reduction in head, or uplift, is experienced. If the depth of the drainage gallery is located and the gallery discharges at the elevation of the tailwater, the magnitude of the uplift downstream of the wells is normally very modest. The uplift, however, is controlled by well spacing, well efficiency, and other seepage control measures, such as grout curtains and cutoffs, in addition to the elevation of the drainage gallery and the elevation at which it discharges. In special cases where the drainage gallery is very deep, i.e., below tailwater, it is possible to actually create a negative uplift on the base of a dam. In such cases some of the seepage pumped from the drainage gallery flows through the foundation to the drains from downstream.

b. Grouting curtains are frequently centered along drainage galleries or tunnels. Remedial or supplemental grouting may be performed from within drainage tunnels. The additional grouting may be performed vertically or in inclined, or sloping, boreholes. Also, should excessive uplift pressures become evident, additional grouting to widen or deepen the curtain may be performed or additional drainage wells may be installed from the gallery to relieve excess pressures.
11-5. Grouting of Foundations and Abutments.

a. The Corps' grouting methods have become less standardized in recent years (Albritton, Jackson, and Banget 1984). Typically, however, a combination of a line of drainage holes and a grout curtain provide an efficient and effective seepage control method. A properly installed grout curtain in the foundation of a structure not only provides a substantial reduction in seepage but also reduces the uplift pressures downstream of the curtain. Conveniently, for the great majority of dam sites, irrespective of type of rock, strike and dip of strata, and faulting conditions, the pervious zone which requires seepage control is relatively shallow and lends itself readily to control by grouting. A comprehensive coverage of drilling methods, as well as grouting methods, is presented in EM 1110-2-3506.

b. Grouting operations in foundations and abutments are not limited to construction of grout curtains. Grouting may be used for foundation repair and filling cavities or voids in limestone or carbonate formations. The requirement for a single- or multiple-line grout curtain is dictated by the condition and integrity of the foundation as determined from preconstruction exploration.

c. Grouting of steep abutment slopes has the potential for causing difficulties and possibly can do more harm than good. Care must be exercised when grouting in abutments to avoid displacements within the rock mass. Even relative low grouting pressures can cause joint opening and decrease the integrity of the abutment.

d. In general the efficiency of a grouting operation for controlling seepage in well graded sediments is proportional to the width of the curtain. In rock foundations and abutments the seepage control accomplished by grouting is not a function of width, however, but is dictated by the effectiveness of sealing seepage paths and open joints identified in the exploration program. The effectiveness of a grouting operation may be evaluated by pre- and post-grouting pressure injection tests for evaluating the water take and the foundation or abutment permeability.


a. Surface treatment of foundations and abutments is essential to ensure intimate contact of backfill materials with sound rock. Once a foundation is exposed by excavation, the method of treatment and potential protection against piping of embankment or abutment materials will be dictated by the conditions encountered. All cavities, or caves, should be cleaned and plugged with concrete, both upstream and downstream from the core trench. All openings, fissures, joints, etc., should be cleaned after excavation and treated with dental concrete, where possible. Treatment with dental concrete stops major deterioration of materials that weather rapidly during construction, and helps to prevent lateral piping of the embankment material into the adjacent foundation.

b. If a well-developed, highly solutioned joint system, or similar condition is encountered in the abutment, thick concrete walls may be placed against the abutment. Prior to placement of concrete walls the abutment face must be set back and cleaned, with presplitting being occasionally required.
Concrete walls offer the advantage of either filling or blocking cavities, affording a reasonable condition for treatment by grouting, and provide an abutment "tie-in" for fill placement with ideal conditions for maximum compaction of the adjacent embankment.
CHAPTER 12
REMEDIAL SEEPAGE CONTROL

12-1. General Considerations. This chapter assumes that a seepage problem with an existing structure has been identified and defined by methods discussed in Chapter 13, or by other observations. The next step is to decide on a remedy, design and install the remedial measure, and monitor its performance to determine if the problem has been satisfactorily addressed. Several factors, including consequences of continued detrimental seepage, the geotechnical environment (embankment, foundation, abutment), and economy, will determine the type and degree of remedial seepage control. Some of the more critical consequences include:

a. Breaching of the embankment or loss of support to structural members due to piping.

b. Breaching of the embankment from slope instability induced by loss of material and/or strength due to seepage.

c. Loss of significant amounts of reservoir water.

d. Maintenance problems or loss of useful areas due to seepage on the downstream slope or areas downstream of the embankment.


a. Factors Affecting Choice of Methods. Several methods of reducing undesirable seepage are discussed in this chapter; most have been previously addressed in Chapters 8-11, which described methods and appropriate settings for each. The remedial designer, while possibly having more advanced technology available than the original designer, must work with existing conditions. The embankment and its foundation, abutments, and seepage control measures may form a complicated structure through which seepage occurs. This can make precise detection and remedial control difficult or impossible. Remedial action may range from continued or additional monitoring to rebuilding or abandonment of the dam. Choice of remedial method(s) will depend on several factors, which include:

(1) Geotechnical environment.

(2) Risk.

(3) Degree of correction required.

(4) Cost.

b. Effects of Methods on Other Structure Elements. The remedial designer must also consider the interplay of the remedial measures with other dam elements. For example:

(1) Effect of excavation for drains, cutoff trenches, slurry trenches, etc., on embankment stability.
(2) Difficulty of tying remedial measure to existing seepage control elements.

(3) Possibility of hydraulic fracturing when grouting.

c. Monitoring. In all cases, pre- and post-remedial monitoring of seepage is essential to determine the effectiveness of remedial action. Since Chapters 8-11 describe control measures in detail, this chapter will just point out primary considerations in the choice of remedial measures and give examples of their use. These examples are provided for general guidance only, since efficient use of remedial measures is very dependent upon geotechnical characteristics of the particular site's as-built configuration, reservoir uses, and pool history.

12-3. Storage Restriction. The most direct method to alleviate a seepage problem is to lower the reservoir and restrict pool levels in order to stop or reduce seepage and its effects. This is often done during problem identification. If piezometer and seepage quantity measurement devices are in place at this time, the effect of this remedy will be experimentally determined. Considerations in storage reduction include:

a. Reduction of downstream inundation area and level should breaching occur.

b. Effects of pool lowering on water supply, flood control, power generation, navigation, recreation, and environment.

Normally, lowering and restriction of the reservoir pool is not an acceptable long-term solution, but this depends on restriction levels and purpose of the reservoir. Care must be taken in lowering the reservoir since rapid drawdown can lead to instability of the upstream slope. Of course, risk of upstream slope failure would normally be a preferred alternative to breaching of the dam and release of a full reservoir.

12-4. Grouting. Grouting is a common, long-used remedy for seepage. Its effectiveness is dependent upon being able to rather specifically locate the leaking area and fill the culprit openings without damage to the embankment. Possible damage includes cracking of impermeable cores or other impermeable areas of the embankment, foundation, or abutments, and clogging of drains. If grouting results in sealing of the foundation just downstream of or beneath the downstream portion of the dam, uplift pressures may increase beneath the embankment or seepage may be forced up into the downstream portion of the embankment. Pore pressure instrumentation should be in place to monitor such changes before grouting begins. This must be considered in design of remedial controls. Because of the many variables in grouting, it is highly desirable to have an experienced contractor and field engineer. In many cases, post-grout drilling may be warranted to determine if the grout has thoroughly penetrated the desired area. Information about grout properties and grouting is given in Chapters 9 and 11. Several case histories follow which provide general examples.
a. 95-ft-High Earthfill Dam (Ley 1974). Upon initial filling, an earthfill dam with a foundation and abutments of volcanic tuffs and breccias exhibited leakage at one of the downstream embankment-abutment contacts and out onto the downstream slope. Inspection revealed leakage from open fractures in the volcanic rock and drains were installed in the areas of seepage. The seepage was stable for a number of years. Subsequent evaluation of the embankment for seismic safety resulted in a need to reduce foundation seepage quantities and piezometric levels within the embankment. A grouting program employed a low viscosity chemical grout in order to penetrate any permeable layers in the embankment where seepage might be occurring. Grout holes were split-spaced for 120 ft along the dam crest from the left abutment. If significant circulation water was lost during drilling, the hole was grouted. Initial spacing was 12 ft with 14 of 23 holes taking low pressure grout (0-5 psi at the collar of the hole). This low pressure was to prevent embankment heave. Most of the take was well into the left abutment with holes spaced as close as 2-1/2 ft and being deepened in stages and further grouted. Gel time varied from 2 to 18 minutes and final depth of holes varied from 24 to 84 ft. Total take was 3,200 gal with seepage being reduced 90 percent, but with little reduction in piezometric levels. Grouting can reduce seepage quantities significantly but still not alleviate high piezometric pressures, particularly in tight or fine-grained materials since any continuous void or pore space can transmit upstream heads.

b. 140-ft-High Earth Dam (Ley 1974). In the left abutment, gypsum had apparently formed in the bedding planes and fractures of folded and faulted shale and siltstone. After water was impounded, leakage, carrying dissolved gypsum, occurred from the abutment. Settlement and gradual increase in seepage also indicated that gypsum was being removed from the formation. Built in 1915, the dam underwent a grouting program from 1930-1933, resulting in placement of about 35,000 cu ft of grout in a series of holes along the dam crest, the left abutment, and at the bottom of the hill forming the left abutment. This program reduced seepage quantities by 75 percent. Approximately 30 years later, over 32,000 cu ft of cement-bentonite grout (colored with iron oxide to distinguish from previously placed grout) was placed in 137 holes to again reduce seepage and replace material removed by solution. Cores indicated good penetration with most seams from hairline to 1/8 in. thick. Seepage was greatly reduced. Other geologic materials may also be dissolved when subjected to seepage. In a similar manner, silt and clay in limestone cavities may also be removed by seepage. Grouting may only be a temporary solution to a seepage problem if solution of a soluble foundation continues after grouting.

c. 70-ft-High Earthfill Dam (Ley 1974). Seepage of 130-140 gal/minute was discovered downstream and attributed to foundation leakage. Installation of drains downstream of the dam allowed collection, metering, and return of water to the reservoir. Drilling for a grouting program, undertaken some years later to reduce seepage losses, revealed loose, sugarlike, decomposed granite 30-40 ft below the dam foundation. The grouting was unsuccessful in reducing seepage. Subsequently, the bottom and right side of the reservoir were covered with an impervious blanket of 40 tons of bentonite mixed with native material. After mixing of the bentonite with native soil to a depth of 3 in., the surface was rolled with a rubber-tired roller. Surface drainage provisions prevented runoff from eroding the blanket during partial pool. Seepage, after
blanketing, decreased 50 percent. In this case, attacking the seepage problem further upstream (at the reservoir) proved more efficient than trying to seal an underlying seepage path.

d. Fontenelle Dam (Gebhart 1974). A 165-ft zoned embankment, Fontenelle Dam, almost failed when a leak of up to 20 cu ft/second developed at the downstream contact with the right abutment. Much of the embankment was eroded before drawdown was effective in stabilizing the embankment. Fortunately, outlet capacity allowed lowering the reservoir 3-4 ft per day. The source of leakage was not specifically determined, but an extensive grouting of foundation rock (calcareous sandstone, siltstone, and carbonaceous shale) was successful in preventing a recurrence of the problem. A 90- by 140-ft cement grout blanket was placed upstream from the original grout cap. Grout curtains were extended beneath the dam beyond the abutments. Over 200,000 cu ft of grout was used.

e. Hills Creek Dam (Jenkins and Bankofier 1972). Hills Creek Dam, constructed by the Portland District, has a maximum height of 338 ft and consists of a central impervious core with gravel and rock shells. Minor seepage occurred near the left abutment during first filling, but decreased with time. Seepage markedly increased in extent and volume after 6 years of normal operation. Vertical drains placed in the downstream shell as an initial remedial measure were not effective in lowering water levels in the downstream shell and seepage continued to increase. An investigation to determine the seepage source continued during the remedial action. Initially it was thought that leakage was through the upstream blanket into the foundation and abutment, but further observations indicated flow was through the core or core-foundation contact. Grouting, which injected 4,500 sacks of cement, most in a 1-1/2:1 mix at zero psi, resulted in elimination of almost all seepage. Four 42-in. bucket auger holes, as well as several smaller borings, were drilled to inspect grouting of the core and foundation. The main source of seepage was at a point along the core foundation contact where a haul road had crossed the abutment. Twelve years later, seepage is still negligible. Frequently, the source of seepage is not obvious. The engineer must consider all possibilities and, after choosing and installing a remedial measure, try to understand what post-remedial monitoring is indicating. The extent of the engineer's knowledge of the foundation, embankment materials, and construction history will greatly influence the accuracy of his analysis of the seepage problem. Often available foundation and construction information will not be adequate and further geotechnical investigation will be required.

12-5. **Upstream Impervious Blanket.**

a. If it is determined that sealing of the reservoir bottom and sides immediately upstream of the embankment will be useful in reducing undesirable seepage quantities and pressures beneath the embankment, an upstream impervious blanket may be employed. If successful and economically feasible, this is one of the most efficient measures since the source of water, the reservoir, is controlled upstream of the embankment and its foundation. This generally requires removal of reservoir water, though some small reservoirs have been sealed by placement of materials through water. Sources of fine-grained material and, in some cases, filter materials are required. The impervious
materials are usually placed on the reservoir bottom. If sloped areas such as the reservoir sides of upstream embankment slope are to be sealed, consideration must be given to protection against wave attack and erosion from runoff. Additionally, fine-grained materials placed on the upstream embankment slope may be removed during drawdown because of low saturated strength and high saturated weight. If seepage can also go through the upstream portion of the embankment and then into the foundation an upstream blanket will be less effective and another remedy may be necessary, e.g., cutoff beneath dam, figure 12-1. The nature of reservoir bottom materials must be considered. Any large voids must be filled with a stable material such as compacted soil, stabilized soil, concrete etc. High gradients will likely exist through the blanket during high reservoir levels, particularly close to the embankment. It may be necessary to place a filter material before placing the blanket to prevent piping of the blanket material into the foundation. The extent of the blanket is determined by analysis and will depend on several factors, including extent of desired decrease in seepage quantities and pressures and blanket material available (quantity and permeability) (EM 1110-2-1913 and Barron 1977). Man-made liners have provided a seal for reservoirs with pervious foundations when fine-grained materials were not economically available. They are usually rather expensive, require relatively smooth surface for placement, and coverings (normally soil) to protect them from puncture in stressed areas and deteriorating exposure to sunlight. Joining of sections is one of the most critical and difficult aspects of man-made liners. Field seams, especially under difficult field conditions and with other than highly experienced personnel, can be an appreciable source of leakage. Quality control of seaming should be strict. One example of the use of an impervious upstream blanket was given in paragraph 12-3c; another is provided below:

b. An impervious upstream blanket connected to a sloping impervious core was placed during the construction of Tarbela Dam on the Indus River in Pakistan (Lowe 1978). The blanket material consisted of sandy silt mixed with a sandy silt angular boulder gravel. The blanket lay over an alluvium of cobble gravel choked with fine sand. The blanket, which was to increase the length of seepage path and not necessarily to reduce seepage quantities, met
the piping criteria, $D_{15}$ (alluvium) $\leq 5D_{85}$ (blanket), Appendix D. As the reservoir emptied after first filling, several sinkholes and cracks were noted in the blanket. Sinkholes ranged from 1 to 15 ft in diameter and 4 to 6 ft in depth. It was felt that uneven settlement during the first reservoir filling caused tension and compression cracks in the blanket which allowed considerable seepage into the underlying sand-choked gravel. In areas where the sand was less dense, the seepage moved the sand down to form a layer in the lower part of the gravel. This created open work gravel just beneath the blanket, and fines from the blanket moved into and through this open layer forming the sinkholes. Sinkholes were filled with filter material and mounded over with blanket material. Typically, the blanket mounds were approximately 15 ft high and extended 30 to 35 ft beyond the sinkhole edge. After filling of the reservoir, sinkholes were located by side-scan sonar and filled with a mixture of filter material and silt from self-propelled bottom dump barges. Each sinkhole generally received 50 barge loads of material. Sinkholes continued to be discovered and covered over another 3-4 years after the initial remedial action. Siltation on the reservoir blanket and filling of sinkholes have reduced seepage about one half.

12-6. **Downstream Berm.** Berms control seepage by increasing the weight of the top stratum so that the weight of the berm plus top stratum is sufficient to resist uplift pressure. If of low permeability, they will reduce seepage, but increase uplift pressures beneath the downstream toe of the dam since they force seepage to exit further downstream of the dam. If pervious, they must be designed as a filter or with an underlying filter to prevent upward migration of line particles from the foundation materials beneath them. Again, a seepage analysis must be made to determine the resisting load required of the berm. Downstream slope stability of the embankment will normally increase because of the resistance to sliding provided by the berm. Huntington District has employed berms as remedial measures at several flood control dams in the Muskingum River flood control system (Coffman and Franks 1982). Similarity of the embankments and environments allowed a standard remedial action for several of the dams at the downstream embankment toe. A 3- to 7-ft-thick pervious blanket of appropriate length is placed over the soft seepage areas at the downstream toe. This adds weight and provides a working platform for installation of relief wells at points of excessive seepage. Another example of a stability berm is given in the Addicks and Barker Dams example, paragraph 12-7a.

12-7. **Slurry Trench Cutoff.** Two major technical considerations in the use of slurry trenches as remedial seepage control measures are (a) the effect on stability of the embankment due to excavation of the trench and the presence of a vertical plane of relatively weak soil (in the case of a soil-bentonite backfill) and (b) tying the slurry trench to other existing or proposed seepage control measures. If a competent upstream blanket exists, the trench may be placed upstream of the embankment and tied to the blanket or may be placed through the dam and any pervious substratum if stability requirements are met. A cement-bentonite backfill may be placed in panels or a concrete wall may be placed in separately excavated elements if an open trench and the relatively weak soil-bentonite backfill are unacceptable because of stability risks. The following experiences with slurry trenches provide general examples of this cutoff type as a remedial measure.
Addicks and Barker Dams, Houston, Tex. (U. S. Army Engineer District, Galveston 1977a; U. S. Army Engineer District, Galveston 1977b; U. S. Army Engineer District, Galveston 1983). Completed in the late 1940's, Addicks and Barker Dams are rolled earth embankments providing flood control in the Houston, Texas, area, with respective maximum heights of 48.5 and 36.5 ft above streambed. Neither normally impound water except in periods of rainfall. The embankments contain some silts and sands, foundations have silt and sand layers, and upstream borrow areas expose the foundation permeable layers. At the time of construction, these conditions were not considered, significant because of the large discharge capability and short detention time of the reservoirs. Residential and commercial development of the downstream local area caused several changes in operating conditions which increased detention time and made the effect of seepage more critical. These included restriction of discharge rates and construction of drainage channels on non-Federal land within 200-300 ft downstream of the center line of the dams which expose the pervious portion of the foundation. Erosion of the drainage channel slopes on the side of the channel nearest the dam and boils in the channel bottom during times of low reservoir impoundment indicated the potential for dangerous seepage conditions during high reservoir levels. Downstream piezometers also indicated a quick response to changes in reservoir levels. This example describes remedial actions at Addicks Dam; actions at Barker Dam were similar. Several remedial measures were considered:

(1) Downstream drainage blanket and stability berm - rejected due to requirement for additional right-of-way and Government responsibility for maintenance of local interest's drainage ditch.

(2) Downstream drainage blanket, stability berm, and relief well system and downstream slurry trench - (relief wells between embankment toe and slurry trench) very positive control (blanket and berm control embankment seepage while wells and slurry trench control underseepage), but very costly, long-term well maintenance required, and all seepage forces would be directed at the embankment toe.

(3) Same plan as (2) except slurry trench replaced with steel sheet pile cutoff - same reasoning as (2) except sheet pile would greatly increase cost.

(4) Slurry trench cutoff through embankment and foundation - a very positive, controlled cutoff for embankment and foundation; no maintenance; all work on Government property; less costly and quicker than other alternatives. For most of the remedial work, alternative (4) was chosen, though for selected lengths of the embankment where they were the best alternative, alternative (1) was used and some relief wells were placed. With a maximum depth of 64 ft and width of 3-5 ft, the slurry trench penetrated 2-4 ft into a relatively impervious clay underlying the pervious foundation materials. Figure 12-2 provides a general cross section of the design. The trench was placed 10-20 ft upstream of the embankment center line with equipment working from a platform established by degrading the upper portion of the embankment. Cemented materials, present in some portions of the excavation, were broken by dropping a 10-ton percussion tool on the cemented layers. Portions of the trench collapsed but were successfully reexcavated. Additionally, small (3-in. diameter) tunnels were encountered in the upstream side of the trench but were
PORTION OF EMBANKMENT THAT MAY BE REMOVED DURING CONSTRUCTION OF SLURRY TRENCH AND REPLACED AFTER COMPLETION OF SLURRY TRENCH

Figure 12-2. Addicks Dams remedial slurry trench for embankment and foundation seepage control (from U.S. Army Engineer District, Galveston)
plugged with cloth. Backfill gradation is shown in table 12-1. For some portions of the project, percent passing for the No. 200 sieve were 15-30 percent. Backfill mixing and transport to the trench were conducted in several ways. Some backfill was batched dry, placed in concrete trucks with slurry added, then mixed and transported to the trench. The higher fines content backfill in some cases proved too sticky to mix in trucks. Mixing was conducted on the ground next to the trench but occasionally excess fines were picked up from the working surface. A concrete mixing pad was used as an alternative though wear from the mixing equipment destroyed the concrete. Excess or unsatisfactory material was deposited in old borrow areas upstream of the embankment to reduce underseepage. In one area, a slurry trench located at the upstream toe of the embankment provided underseepage control while a downstream berm provided embankment stabilization. The berm of sandy clay had permeability characteristics similar to the embankment and provided a 1V on 8H slope. Several of the discharge conduits which suffered from seepage and piping were resealed, after cleaning, with ethafoam backer rods and a polyurethane sealant. Where the sealant would not adhere to the concrete, joints were talked with oakum soaked with a grouting compound. Well screens were placed in weep holes to prevent loss of soil, and relief wells with submersible pumps were installed. For certain portions of Barker Dam, use of an upstream clay blanket and a downstream stability berm (1V on 8H) was more cost effective than a slurry trench. There was intermittent surface exposure of pervious foundation materials and a source of CH materials for the blanket was available within the reservoir. Prior to placement of the blanket, ponded water and soft surface materials were removed. The blanket was placed in 8-in. layers and compacted with tamping rollers at natural moisture content.

Table 12-1. Backfill Mix for Slurry Trench, Addicks Dam

<table>
<thead>
<tr>
<th>Sieve Size or Number (U. S. Standard)</th>
<th>Percent Passing by Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 in.</td>
<td>100</td>
</tr>
<tr>
<td>1-1/2 in.</td>
<td>95 to 100</td>
</tr>
<tr>
<td>3/4 in.</td>
<td>80 to 100</td>
</tr>
<tr>
<td>No. 4</td>
<td>55 to 100</td>
</tr>
<tr>
<td>No. 10</td>
<td>40 to 80</td>
</tr>
<tr>
<td>No. 40</td>
<td>18 to 45</td>
</tr>
<tr>
<td>No. 200</td>
<td>10 to 25</td>
</tr>
</tbody>
</table>

(a) From U.S. Army Engineer District, Galveston.

Though not yet severely tested, the control measures have performed satisfactorily based on the following observations:
a. Foundation downstream piezometers do not respond to reservoir levels experienced so far.

(b) Phreatic surface has been raised upstream of the slurry trench.

(c) No embankment seepage, but there have been no significant pools.

(d) Settlement plates indicate no significant settlement of the slurry trench. Though the restored embankment has cracked in the area of the trenches, inadequate compaction of the embankment fill is considered the cause.

b. Wolf Creek Dam, Ky. (Fetzer 1979). Constructed in the 1940's, Wolf Creek Dam is a 200-ft-high combination earthfill and concrete dam founded on limestone containing shale and solution cavities. During excavation of a 10-ft-wide cutoff trench, several interconnected solution cavities were discovered in the limestone. These were backfilled for a short distance with impervious material, and a 50-ft-deep single-line grout curtain was placed beneath the bottom of the cutoff trench. In 1967, muddy flow was observed in the tailrace, a small sinkhole developed near the downstream toe, and wet areas existed near the downstream toe. In 1968, a larger sinkhole developed (13 ft wide, 10 ft deep) and drilling revealed solution features running perpendicular and parallel to the dam axis. It was concluded that reservoir water was passing beneath the cutoff trench. Grout lines were placed along the dam axis near the embankment-concrete contact and downstream of this area. During 1971-1972, an overall assessment of the seepage problem was made since the remedial grouting had only addressed about 200 ft of the 4,000-ft embankment portion of the dam. A diaphragm concrete cutoff wall was considered the best solution because it could be installed without draining the reservoir, a very costly operation due to reservoir use. Explorations, which included borings spaced on 3.1-ft centers along the axis of the wall (parallel to the dam axis), defined the depth and length of the wall. Depth was 10 ft below the lowest indication of solution activity (maximum depth 278 ft) and length was 2,239 ft. In 1974, a request for technical proposals resulted in seven proposals with two acceptable. In the second stage, a bid invitation was issued and an award was made for a wall in the area of the switchyard and 989 ft of the wall along the dam axis. The award in 1975 was followed by a second competition and an award in 1977 for the remaining 1,250 ft of the axis wall. The wall consists of alternate cylindrical primary elements and connecting secondary elements installed using bentonite slurry, figure 9-14. Primary elements are 2.17-ft-diam steel casings filled with tremied concrete (see table 9-8 for mix proportions). Weak cement grout fills the volume between excavation walls and the casing. A 25-ft-deep core hole was drilled beyond the bottom of each primary element to explore for cavities and was pressure-tested and grouted prior to the placement of a closed-end primary casing. The primary element was required to set for a minimum of 20 days before excavation of the secondary element which is also filled with tremied concrete. Frequent piezometer readings (as often as every 4 hours) were made during construction to determine the hydraulic condition of the embankment and foundation and warn of any potentially critical seepage conditions. Excavation and drilling were closely monitored to observe any drill rod drops or mud losses. Sealers and reserve mud, constantly on hand, provided for emergencies. Grout takes around
the primary casings and volume of concrete used in the secondary elements were closely monitored as was the embankment in general. Efficient management of a large number of observations was necessary to determine the current condition of the dam. The lack of major losses of slurry, grout, or concrete during construction was probably due to the densely spaced borings and grouting done during the earlier exploration program. Wall construction, completed in 1979, took approximately 4 years and two construction contracts. Subsequent piezometric levels indicate the wall is a successful seepage barrier.

c. Camanche Dike 2, California (Anton and Dayton 1972). One of several earthfill dikes containing Camanche Reservoir, Dike 2, is a zoned earth embankment about 70 ft high founded on alluvium containing an upper 20-ft strata of clayey sand underlain by layered silty-to-fine uniform sand stratum. The underlying sand stratum varies in permeability with its lower portion containing gravel. Original construction involved extending the core horizontally to the upstream toe and discing and compacting the top of the alluvium to 1,000 ft upstream of the dike axis. This was expected to provide acceptable seepage conditions, while providing the option of tying an upstream cutoff through the alluvium to the core if operating underseepage conditions were intolerable. After reservoir filling, underseepage proved extensive and flowed over downstream property. Lowering of the reservoir reduced the seepage and revealed holes in the compacted alluvium upstream of the embankment. Several seepage control methods, including upstream impervious blanket, grout curtain, relief wells, downstream drains, sheet piles, and others, were considered. Evaluation of the options resulted in choosing an upstream slurry trench. This method provided a positive cutoff, minimized piping potential, allowed retention of a partial reservoir, and was the least expensive of positive cutoff methods. Placed 50 ft upstream of the upstream berm toe, the 1,660-ft-long slurry trench, 8 ft wide, extended through the alluvial materials to a maximum depth of 95 ft. Backfill specifications required a 4-in. slump and a gradation as shown in Table 12-2. An 8- to 11-ft-deep sandy clay blanket protected by a 1-ft-thick cobble and gravel cover connected the slurry trench to the horizontal core extension. Excess slurry was blended into the top portion of the blanket to decrease blanket permeability. Since slurry trench placement, downstream piezometers reflect decreased influence of reservoir levels with only very small seepage flows at high, prolonged reservoir levels. Two potential sources of seepage are the somewhat pervious bedrock formation which the slurry trench is keyed into and the sandy clay blanket connection between the slurry trench and extended core. Connection of the 8- to 11-ft-deep blanket to the slurry trench after placement of the slurry was difficult and may allow reservoir leakage into the alluvium. Placement of the blanket or a partial thickness prior to slurry trench construction was recommended. This would provide a platform for construction of the slurry trench and allow a more secure attachment of the trench to the blanket. This procedure has been standard practice on many subsequent slurry trench projects.

12-8. Relief Wells. Though Chapter 9 describes design and installation of relief wells, additional factors must be considered when relief wells are used for remedial seepage control. Relief wells can relieve excessive uplift and potential piping when pervious layers are overlain by relatively impervious strata by providing controlled release of relatively large volumes of water. Relief wells, as compared with cutoffs, allow loss of reservoir water and
Table 12-2. Backfill Mix for Slurry Trench, Camanche Dike 2 \(^{(a)}\)

<table>
<thead>
<tr>
<th>Sieve Size or Number (U. S. Standard)</th>
<th>Percent Passing by Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 in.</td>
<td>100</td>
</tr>
<tr>
<td>3 in.</td>
<td>80 to 100</td>
</tr>
<tr>
<td>3/4 in.</td>
<td>60 to 100</td>
</tr>
<tr>
<td>No. 4</td>
<td>40 to 80</td>
</tr>
<tr>
<td>No. 30</td>
<td>20 to 60</td>
</tr>
<tr>
<td>No. 200</td>
<td>10 to 30</td>
</tr>
</tbody>
</table>

\(^{(a)}\) Courtesy of American Society of Civil Engineers.\(^{135}\)

require proper handling of discharge flows and periodic maintenance. Flooding and erosion from well discharges must be prevented. Wells may be installed quickly with a minimum of downstream right-of-way and, in many cases, without reducing reservoir levels. If high uplift is present, boring and installation may be difficult requiring extra measures to keep the hole open and stable until the screen and filter are installed.

12-9. Drainage of Downstream Slope. Seepage emerging on or at the toe of the downstream slope will normally be controlled by one of the methods previously mentioned. Expedient installation of filter materials and a toe drain can help prevent piping of embankment and foundation materials and may increase embankment stability, but will not normally reduce seepage quantities. If seepage is confined to a small area or areas, horizontally drilled drains may help control the problem (Royster 1977). Horizontal drains of slotted pipe normally do not have a filter envelope and would generally be used for "nuisance" seepage or as an expedient measure until a more permanent solution could be installed.
13-1. General Considerations.

a. Before seepage control measures are implemented, site characterization by thorough exploration and testing is needed to determine if the seepage control measures will serve the intended purpose. Knowledge of the in situ site conditions along with the purpose of the seepage control measure and its physical dimensions will help determine the overall number and the placement of monitoring devices. Several factors which can be monitored that can lead to a conclusion regarding the safety of a dam are: (1) progressive increase in the volume of seepage flow, (2) removal of solids by the seepage, (3) increased uplift pressures or locally depressed gradients, and (4) soft or wet areas on the downstream embankment.

b. Monitoring the performance of seepage control measures can lead to a collective conclusion drawn from several measurements. The most common and easiest monitoring is to rely on visual observations along with careful surface inspections at predetermined intervals. Another type of monitoring which should be completed before construction is the installation of piezometers, observation wells, and drainage collection systems to determine a site dependent pattern of behavior. Finally, the actual structure should be monitored by the installation of a site specific network of piezometers, observation wells, and drainage collection systems with flow measurements designed for the anticipated seepage problems. A regular review of the data collected will generally detect major changes between subsequent readings but equally as important are the long-range trends manifested by steady changes or intermittent surges.

c. If it is determined during the monitoring process that a possible problem exists, an expanded instrumentation program may be needed. This could include more piezometers, relief wells, etc., and a more extensive analysis of the seepage water; i.e., both a physical and chemical analysis of the sediment and water including temperature, salt content, and resistivity which could be compared with samples from the embankment and possible seepage sources. According to the complexity of the problem and/or the economics versus safety involved a group of other studies could be added including, but not limited to: resistivity and spontaneous potential of the embankment and foundation, dye tracing, infrared (aerial or portable ground based), and seepage acoustic emissions. In most cases, the scope of the monitoring program will be determined by the economics involved.


a. Foundation. To determine the performance of seepage control measures, a pattern of behavior should be established prior to and during construction where long-term trends can be related to design or seepage conditions. Piezometers should generally be installed in all compressible foundation soils, the number being dependent on the extent and thickness of the strata. If possible, foundation piezometers should be installed-in the sections selected for embankment piezometers and should extend beyond the upstream and downstream toes a
distance equal to the expected migration of pore pressures. An effective piezometer installation plan should convert preconstruction piezometers into postconstruction seepage monitoring piezometers for an effective continuity of data. Shown in figure 13-1 is an example of a piezometer installation with the embankment resting on a compressibility foundation (Chapter 9) and an impervious core cutoff through a sand and gravel layer. Effectiveness of the cutoff and the presence of uplift pressures can be evaluated. Shown in figure 13-2 is an embankment with an impervious core that intercepts a pervious soil and rests on the top of rock (Chapter 11) and again the effectiveness of the cutoff and/or the integrity of the rock can be evaluated. Shown in figure 13-3 is an embankment founded on a thin impervious top stratum underlain by a deep zone of pervious material (Chapter 9). In this case a cutoff was impractical and seepage pressures are simply monitored beneath the dam to determine time effects on the control measures while the effectiveness of relief wells downstream is determined with foundation piezometers. An effective monitoring system should also include piezometers in the abutments to determine the effectiveness of drains and/or of the embankment-abutment interface which could include grout curtain cutoffs (Chapters 10 and 11). Also artesian flows may exist in the abutments and need to be monitored. Shown in figure 13-4 is a piezometer installation that is used to monitor a grout curtain cutoff and cut-slope drains. Remedial seepage control measures, discussed in Chapter 12, might require additional piezometers to monitor both the installation and the effect of the new measures.

b. Embankment. As discussed in Chapter 8 three methods for seepage control in embankments are: (1) flat slopes with or without drains, (2) embankment zonation, and (3) vertical (or inclined) and horizontal drains. An embankment with flat slopes (as defined in Chapter 8) constructed of impervious material, and which has infrequent high reservoir levels, should have only enough piezometers in the embankment to establish the phreatic surface. A typical example is shown in figure 13-5. To monitor seepage control measures in a zoned embankment (Chapter 8) the number and spacing of piezometers depend not only on the height of the dam but also on the material properties of the zones. The core must be monitored to determine the phreatic

Figure 13-1. Example of foundation piezometer installation of Surry Mountain Dam (from EM 1110-2-1908)
Figure 13-2. Piezometer installation for dams on pervious foundations, Deer Creek Dam (from EM 1110-2-1908)
Figure 13-3. Piezometer installation with a thin impervious topstratum and relief well systems, Enid Dam (from EM 1110-2-1908)
Figure 13-4. Piezometer installation for dam abutments having grout curtains or cut-slope drains, Applegate Lake Dam (from U. S. Army Engineer District, Portland)
Figure 13-5. Estimated seepage for Aubrey Dam, Texas (from U. S. Army Engineer District, Fort Worth)
surface and in situ permeability. Using this information, cracking potential can be estimated. On either side of the core there can be filters, transition zones, random zones, outer shells, and blankets (Chapter 8). In general, the different zones should increase in permeability outward and should have piezometers in each zone. During rapid reservoir drawdown, the upstream piezometers would detect excess pore pressure buildup while during conservation and high pools, all the piezometers would indicate the effectiveness of the zones and check design values. A typical installation in a zoned embankment is shown in figure 13-3. In a zoned embankment the piezometers located near the upstream core face can be instrumental in determining the development of cracking in the core (Vaughan et al. 1970). Depression of the piezometric elevation in a localized area (cone of depression) can indicate velocity head loss which would indicate leakage through the core. Remedial work for Baldershead Dam (Vaughn et al 1970 and Lovenburg 1974) included placement of a large number of piezometers near the upstream face of the core (figure 13-6) which were successful in indicating piezometric depressions.

c. Drains. The purpose of vertical (or inclined) and horizontal drains is to control seepage either through the embankment or beneath the dam (underseepage). A vertical (inclined or horizontal) drain in the embankment may be used as a filter to prevent material from eroding from the core and/or as a method of collecting seepage exiting from horizontally stratified soil layers. Enough piezometers should be installed in the drain to determine if the seepage is coming through the embankment material or if it is underseepage, figure 13-7. If the horizontal drains intercept underseepage which in turn is drained by lateral drains, piezometers should be placed on either side of the laterals to determine their effectiveness, figure 13-8. Long-term trends (pressure buildup or depression) detected in the drains not directly related to the reservoir level could indicate either clogged drains (pressure buildup) due to embankment or foundation material moving into the drains or piping and erosion (pressure depression) due to material moving into pipe drains, high permeability zones, or into fractured rock. Toe drains are effective in collecting seepage and preventing saturated areas along the downstream toe. Piezometers in or near toe drains would only be effective when a downstream blanket has been added and uplift pressures need to be measured. Drainage galleries and tunnels are used mostly in abutments in the United States to intercept and control seepage in fractured rock. Drainage tunnels along extensions of the dam's axis or in downstream abutment areas serve to collect seepage. Piezometers located near the drainage tunnels would indicate the effectiveness of the tunnel. Cut-slope drains can be used to intercept seepage and collect drainage along abutment slopes while piezometers placed both upstream and downstream of the drains can determine effectiveness of the collection system, figure 13-9.

d. Downstream Areas. Seepage can migrate beyond the embankment toe particularly in clay shale or fissured formations. Geologic site characterization in most cases will determine the need for piezometers downstream of the toe but if there is any doubt they should be installed in the questionable formations 50 to 150 ft beyond the toe, figure 13-1. Piezometers should also be installed downstream of the outlet works, spillway, and stilling basin if they extend well beyond the toe and are not close to other piezometers.
Figure 13-6. Layout of Baldershead Dam piezometer installation in upstream shale fill (courtesy of British Geotechnical Society)
Figure 13-7. Piezometer installation for dams with through seepage and underseepage (from U. S. Army Engineer District, Baltimore)
Figure 13-8. Piezometer installation for dams with lateral drains (from U. S. Army Engineer District, Baltimore)
Figure 13-9. Drain and piezometer installation, Applegate Lake, Oregon (from U. S. Army Engineer District, Portland)
e. Near Relief Wells. If a relief well system has been installed, pore pressures should be checked in the vicinity to evaluate the efficiency of the system. Piezometers should be located both upstream and downstream of the relief wells and should intercept the stratum being drained, figure 13-3. If there is a line of wells, piezometers should also be installed generally at the midpoint between the wells or at the point expected to have the highest pore pressures.

f. Spillways, Stilling Basins, and Outlet Works. Underseepage control beneath the stilling basins of spillways and outlet structures founded on pervious foundations is generally provided by drainage blankets supplemented in the case of stratified foundations by deep well systems (figures 13-10 and 13-11). Drainage blankets extend beneath the chute slabs, if necessary. As shown in figure 13-10, piezometers should be installed to check the effectiveness of the drainage blanket and relief wells, and to check pore pressure beneath the outlet channel and against the stilling basin walls. Piezometers are used to check pore pressures occurring below the relief well system (figure 13-10). Generally, a sheet pile wall, with a minimum penetration of 15 ft, is installed along the downstream toe of the stilling basin to control piping and a piezometer installed downstream of the wall is needed to determine the effectiveness of the wall.


a. Weirs. Seepage flow measurement is an important parameter of dam performance. Most installations have used a relatively simple weir, measuring the seepage over brass or stainless steel 90-deg V-notches like the collection system shown in figures 13-12 and 13-13. A number of weirs can be installed in drainage galleries to determine flows from different sources, i.e., left or right abutment, dam underseepage, and total seepage. A certain amount of sediments will settle out just upstream of the weir which is important if the sediment load is not continuous and occurs between visual inspections. If the seepage is exiting downstream of the dam and outside of a drainage collection system, a weir pond can be formed in conjunction with the V-notch weir for the specific purpose of determining long-term sediment content in the seepage.

b. Flumes. A flume is a short rigid-walled channel designed to constrict the flow and so give rise to critical velocity. A single measurement of water level is sufficient to measure discharge at critical velocity. The most commonly used flume for seepage measurements is the Parshall flume as shown in figure 13-14. Empirical charts for flow discharge have been developed for specified flume dimensions (Bureau of Reclamation 1967). The flume can be fabricated and placed in seepage flow that has been channelized. This method is a relatively rapid and simple way to obtain precise flow measurements.

c. Relief Wells. Relief wells, as the name implies, are widely used to relieve pressures and control seepage through pervious strata beneath earth dams, spillways, and outlet works. A thorough knowledge of the geologic conditions and characteristics of the soils at the dam must be available to design a system as part of initial construction or as remedial work. To be effective, the well must flow but must not allow the loss of foundation
Figure 13-10. Piezometer installation for dams with outlet structures (adapted from U. S. Army Engineer District, Vicksburg)
Figure 13-11. Piezometer installation for spillways (adapted from U. S. Army Engineer District, Vicksburg)
Figure 13-12. Installation of drainage system with weirs (from U. S. Army Engineer District, Portland)
Figure 13-13. Typical weir installation (from U. S. Army Engineer District, Portland)
Figure 13-14. Modified Parshall flumes of 6- to 65-second-foot capacity (after Bureau of Reclamation)

Figure 13-14. Modified Parshall flumes of 6- to 65-second-foot capacity (after Bureau of Reclamation)

material which could lead to piping or erosion failure. A frequent use of relief wells is near the downstream toe of a dam to control seepage through a pervious zone, figure 13-13. A toe drain is used to collect the flow while weirs in the drains or manometers on the wells can determine the quantity of flow. Relief wells can be pumped to determine the effect on surrounding ground water and the permeability of the soil near the well. In many cases where dams are built on jointed or fractured rock, grout curtains are used as seepage cutoffs. It has been found in some cases that grout curtains do not appreciably affect the uplift pressures downstream of the curtain and that a series of drainage wells is a more effective tool to reduce uplift pressures when the volume of seepage loss is not a problem and when the high cost of grouting is hard to justify (Casagrande 1961). Relief wells have been successfully used when the pervious strata is too deep and wide-ranging to effectively use any type of positive cutoff. Relief (drainage) wells are used beneath spillway and outlet works slabs to relieve excess pressure in the rock, the underslab drains, or a pervious strata. An example of a group of wells designed to relieve pressure under an outlet works stilling basin is shown in figure 13-10. Relief wells can be used in rock abutments to intercept seepage and to control artesian flow.

d. Seepage Outlets. Monitoring seepage outlets downstream of the dam is handled according to the present severity of the problem or to future associative problems if the seepage worsens. A small wet zone near the toe might require only routine visual examination as would a small trickle from a rock abutment. If there is sufficient seepage to measure, an effort should be made to estimate the volume with a container and a stopwatch and to note sediment content. Operation and maintenance personnel should be trained to
observe the situation especially during high reservoir levels. All seepage outlets should be monitored to determine long-term trends. Sand boils developing downstream require immediate attention. An estimate of the pore pressure involved can be determined by sandbagging around the boil to measure the height of water rise. If the seepage exiting at the toe or abutment is severe and will require remedial work but the dam is not in imminent danger of failing, collection systems must be designed as a pattern develops. Temporary control measures such as toe drains and surcharge berms might be installed with weirs to establish the severity and the trends of the seepage for use in design of remedial work. Any changes noted during monitoring, i.e., volume change, sediment load change, etc., should be considered important. After the remedial work has been designed it may include any number of the monitoring systems discussed previously in this chapter. If the seepage is exiting from a drain system that is not being monitored internally, visual monitoring should continue at specified intervals and during heavy runoffs and high reservoir levels.

13-4. **Seepage Water Analysis.**

a. **Physical Analysis.** Physical analysis of the seepage could include information on suspended solids, temperature contours, and water resistivity values. This information may be obtained from physical testing of samples or by remote testing techniques.

(1) The amount of suspended solids in the seepage is an indication of material movement and piping. Although there are obvious problems with muddy or turbid flow, seepage which appears clear may often carry small amounts of suspended solids that would be detected by occasional samples and analysis. Sediment traps built in conjunction with manholes and weirs can be used to indicate the amount of suspended sediment and to obtain samples for chemical analysis.

(2) Several different methods of measuring temperature are designed to help locate seepage areas not yet visible and to trace seepage from its origin to its exit. One remote sensing method (U.S. Army Engineer District, Los Angeles 1981) is an aerial survey which includes any or all of the following: (a) color photography, (b) color infrared photography, (c) thermal infrared, and (d) color oblique imagery. The basis for the study is that different materials (wet or saturated versus dry) possess different heat absorption rates; therefore, heat radiation rates will differ. Since the specific heat of water is higher than soil or rock, a warm zone in a known seepage area is a suspected seepage outlet. This method is intended for large areas but smaller areas can be covered by thermal infrared using portable hand-held units (Leach 1982). A second method of thermal monitoring (U. S. Army District, Los Angeles 1981) is to physically place an array of gages in and adjacent to the embankment and measure the diurnal temperature (temperature below the reach of the surface but within the annual temperature zone). A temperature fluctuation is interpreted as seepage related and becomes the basis for further study in that zone. A vertical thermal contour can be made in present open system piezometers or in remedial planned piezometers, again with temperature fluctuations or inversions interpreted as seepage related (U. S. Army Engineer District, Los Angeles 1981 and Leach 1982). These data are interpreted or
expanded by determining the temperature stratification of the reservoir, the temperature of all known seepage sources, and the temperature at the seepage exits.

(3) Seepage can be monitored by the physical detection of tracer elements such as dye and isotopes that have been introduced upstream of the seepage exit either in the reservoir or a piezometer close to the suspected seepage path. The dye selected should have a favorable absorption and decay rate and should meet state water quality control requirements. Samples taken from piezometers, drains, and downstream exit points are monitored using an instrument capable of measuring parts per billion, e.g., a fluorometer for fluorescein dye. By recording time of arrival and concentrations, interpretations can be made as to the source of the seepage and the permeability of the strata along the path of seepage. Environmental isotopes are also traced by obtaining samples which can be measured by a mass spectrometer for oxygen -18 and deuterium and by the low level counting system for tritium.

(4) Another physical property of the seepage that can be measured is its conductance or resistance. Resistance which would be defined in the field by a resistivity survey is a measure of the ability to resist current flow through the seepage, a factor that is altered by the introduction of salt compounds, graphite, etc. A thorough sampling program from all possible sources of seepage, all seepage exits, and all available piezometers can produce a group of resistivity values that is an important tool in defining seepage sources and possible paths. Using the geologic profile for the site, and by comparing individual resistivity values or by comparison against a range of known values (Telford et al. 1976), an interpretation as to the source of the seepage and the strata through which it travels can be made.

b. Chemical Analysis. To monitor and interpret the chemical composition of seepage requires a thorough knowledge of the surrounding geology or chemical analysis of samples in the different strata. If possible, a pre-construction chemical analysis should be conducted on all water sources and on any formation that might contribute minerals or salts or that might affect acidity or alkalinity.

(1) One important chemical property would be the salt concentrations in the seepage. In this case to determine correlations, reservoir and groundwater concentrations are essential along with the mineral content of the area, e.g., water flowing through limestone would generally increase in chloride concentration. Minerals such as feldpoid sodalite and apatite (Turkish National Committee 1976) or caliche in volcanic regions (U. S. Army Engineer District, Los Angeles 1981) can release chloride ions into the water. Interpretation of chloride concentrations and its long-term trends can help determine the relative length of seepage paths (shallow or deep seated), the extent of the leaching of the formation (whether concentrations are constant, increasing, or decreasing), and the source of the seepage (comparable concentrations). Interpretation is sight-dependent and any of the above or possibly other conclusions may be reached.

(2) Another chemical property needed for interpretation is the mineral content of the seepage. Mineral concentrations of calcium and magnesium
bicarbonate containing dissolved carbon monoxide increase with length of flow through a limestone formation while flow through some clay minerals may increase concentrations of calcium and magnesium. Again, interpretation is site dependent and requires a thorough knowledge of the existing conditions.

(3) Stiff diagrams, as shown in figure 13-15, are a graphical method of presenting the anions and cations that are dissolved in the water (Hem 1970). The Stiff method uses ions plotted in the same sequence to give an irregular polygonal shape or pattern. In tracing the movement of seepage water, the Stiff patterns are plotted on a map of the site for various locations downstream of the dam and the reservoir. The Stiff patterns may yield information regarding the path(s) of seepage from the reservoir, prior land uses downstream of the dam (feedlot, septic field, etc.), and type of formation (gypsum, dolomite, etc.).

13-5. **Remote Sensing Methods.**

a. **Resistivity and Spontaneous Potential.** As part of a total seepage study, resistivity and spontaneous potential methods have been used successfully for seepage delineation in soil and rock (Cooper and Bieganousky 1978; Cooper, Koester, and Tranklin 1982; and Koester et al. 1984). Resistivity surveys used as part of a seepage study help identify possible zones of high moisture as a function of depth and location. After the surveys are correlated to previous borings or geologic information, new borings are placed in the seepage flows. Spontaneous potential surveys are used to detect negative D. C. voltage anomalies in the surface electrical field which have been found to indicate zones of seepage flow (Cooper, Koester, and Tranklin 1982; and Koester et al. 1984). Although flow is indicated, depth to flow can not be determined for a given anomaly.

b. **Photography.** Methods using color, color infrared, aerial and ground base thermal infrared, and color oblique photography were discussed previously in paragraph 13-4.a.

c. **Refraction Seismic Surveys.** Seismic surveys can be used indirectly in a seepage study by providing a bedrock profile that can be used as an aid in determining the location and depths of observation wells, piezometers, and relief wells. It is a quick and inexpensive method for obtaining subsurface profiles.

d. **Seepage Acoustic Vibrations.** Acoustic emissions are the noises generated whenever a material deforms or possibly by seepage whenever there is turbulent flow against and around a casing (Koerner, Lord, and McCabe 1977). The technique has been used to detect seepage by placing an accelerometer on a waveguide that extends to the bottom of a borehole and recording the vibrations present (Koerner, Lord, and McCabe 1977; and Leach 1982). Increases in emissions activity are interpreted as seepage flow. A similar technique that consists of lowering an acoustic microphone down into a reservoir has been used to detect leakage on the upstream asphalt-covered face and in the reservoir itself (Coxon and Crook 1976). One disadvantage would be high background noise levels.
Figure 13-15. Stiff diagram used to graphically present anions and cations in seepage water (from Hem 1970)
CHAPTER 14
INSPECTION, MAINTENANCE, AND REHABILITATION OF
SEEPAGE CONTROL MEASURES

14-1. Introduction. Proper functioning of seepage control features requires adequate maintenance, inspection, and, if necessary, rehabilitation. Some seepage control methods such as relief wells are in the best condition they will ever be the day they are installed and developed, while others such as soil-bentonite slurry trench cutoffs may increase in effectiveness with time. All seepage control features must function effectively for the life of the dam or be rehabilitated or, if necessary, replaced. The effectiveness of some seepage control methods, such as the toe trench drain, may be directly observed. For other seepage control methods, such as cutoffs, performance monitoring (see Chapter 13) is essentially the only means of determining the degree of effectiveness.

14-2. Inspection. The procedure for periodic inspection and continuing evaluation of dams is given in ER 1110-2-100. Procedures for reporting evidence of distress in dams are given in ER 1110-2-101. Procedures in these two ER's are often supplemented by Division Regulations as well. Details concerning the monitoring performance of seepage control measures are given in Chapter 13. The first general field inspection for new earth and rock-fill dams is carried out immediately after topping out the embankment. The initial inspection of concrete dams is accomplished prior to impoundment of reservoir water. The second inspection for earth and rock-fill and concrete dams is made at a reasonable stage of normal operating pool but no later than one year after initial impoundment has begun. Subsequent inspections will be made at one-year intervals for the next four years, at two-year intervals for the following four years, and then may be extended to every five years if warranted. The periodic inspections provide the opportunity for a group of specialists to critically examine a project for existing and/or potential problems, to recommend remedial action or changes in instrumentation, and to direct the attention of the operating personnel toward the significant and critical features of a project. However, the occasional inspection cannot take the place of daily observations required to detect potentially dangerous problems at an early and repairable stage. Table 14-1 outlines the inspection, instrumentation, maintenance, and rehabilitation of seepage control facilities. Some seepage control methods such as embankment zonation, cutoffs, and upstream impervious blankets are not amenable to visual inspection. Other methods such as flat slopes downstream of the dam and downstream seepage berms are most accessible and should be inspected daily during periods of full reservoir pool to ascertain that they are functioning properly in controlling seepage. Other seepage control facilities should be inspected on a regular schedule as shown in table 14-1.

14-3. Maintenance. Timely performance of maintenance on seepage control facilities is required for the facilities to perform satisfactorily. Some seepage control methods such as flat slopes downstream of the dam and downstream seepage berms are accessible and require maintenance. Other seepage control methods such as embankment zonation, cutoffs, and upstream impervious blankets do not require maintenance. However, maintenance is required on the instrumentation used to evaluate the degree of effectiveness of all seepage
### Table 14-1. Inspection, Instrumentation, Maintenance, and Rehabilitation of Seepage Control Facilities

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<thead>
<tr>
<th>Seepage Control Facility</th>
<th>Frequency (a)</th>
<th>Type of Observation</th>
<th>Frequency (a)</th>
<th>Type of Measurement</th>
<th>Type of Maintenance</th>
<th>Method of Rehabilitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat slope without drains</td>
<td>Daily</td>
<td>Wet spots, sloughing, erosion</td>
<td>Monthly</td>
<td>Surface monuments</td>
<td>Fertilising, mooving, filling</td>
<td>Extend slope thickness and length</td>
</tr>
<tr>
<td>Embankment core</td>
<td>--</td>
<td>--</td>
<td>Varies (b)</td>
<td>Piezometer</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Vertical and horizontal drains</td>
<td>Weekly</td>
<td>Turbidity and discharge rate</td>
<td>Varies (b)</td>
<td>Piezometer</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Horizontal drain</td>
<td>Weekly</td>
<td>Turbidity and discharge rate</td>
<td>Varies (b)</td>
<td>Piezometer</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Cutoff</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Compact backfill trench</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Slurry trench</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Soil-bentonite</td>
<td>--</td>
<td>--</td>
<td>Varies (b)</td>
<td>Piezometer</td>
<td>--</td>
<td>Install short parallel adjacent trench</td>
</tr>
<tr>
<td>Cement-bentonite</td>
<td>--</td>
<td>--</td>
<td>Varies (b)</td>
<td>Piezometer</td>
<td>--</td>
<td>Install short parallel adjacent trench</td>
</tr>
<tr>
<td>Concrete well</td>
<td>--</td>
<td>--</td>
<td>Varies (b)</td>
<td>Piezometer</td>
<td>--</td>
<td>Grout adjacent to wall</td>
</tr>
<tr>
<td>Upstream impervious blanket</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Downstream seepage berm</td>
<td>Daily</td>
<td>Seepage, boils, erosion</td>
<td>Monthly</td>
<td>Surface monuments</td>
<td>Fertilising, mooving, filling</td>
<td>Extend berm thickness and length</td>
</tr>
<tr>
<td>Toe trench drain</td>
<td>Weekly</td>
<td>Turbidity and discharge rate</td>
<td>--</td>
<td>--</td>
<td>Check periodically (e)</td>
<td>--</td>
</tr>
<tr>
<td>Relief wells</td>
<td>Varies (b)</td>
<td>Flow rate and sand infiltration</td>
<td>Varies (b)</td>
<td>Piezometer</td>
<td>Check periodically (f)</td>
<td>Rehabilitate wells</td>
</tr>
<tr>
<td>Concrete galleries</td>
<td>Weekly</td>
<td>Turbidity and discharge rate</td>
<td>--</td>
<td>--</td>
<td>Check periodically (g)</td>
<td>--</td>
</tr>
<tr>
<td>Upstream impervious blanket</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Downstream filter layer</td>
<td>Weekly</td>
<td>Turbidity and discharge rate</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Relief wells</td>
<td>Varies (b)</td>
<td>Flow rate and sand infiltration</td>
<td>Varies (b)</td>
<td>Piezometer</td>
<td>Check periodically (f)</td>
<td>Rehabilitate wells</td>
</tr>
<tr>
<td>Drainage blanket</td>
<td>Weekly</td>
<td>Turbidity and discharge rate</td>
<td>Varies (b)</td>
<td>Piezometer</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Relief wells</td>
<td>Varies (b)</td>
<td>Flow rate and sand infiltration</td>
<td>Varies (b)</td>
<td>Piezometer</td>
<td>Check periodically (f)</td>
<td>Rehabilitate wells on unwatering of structure</td>
</tr>
</tbody>
</table>

(a) Schedule to be followed upon attainment of full reservoir pool. The schedule during initial filling will be determined during the initial periodic inspection.

(b) Observations should be made one week after maximum reservoir level and at three subsequent falling reservoir stages.

(c) Varies depending upon potential sedimentation problems, impact on project performance, and impact on stream system (see EM 1110-2-4000).

(d) Filling or falling head test should be conducted on piezometers annually before the storage season (see EM 1110-2-1908, Part 1).

(e) Catch basins, manholes, ditches, drainage pipe, and weirs should be cleaned, as a minimum, in the fall in preparation for the winter season, and again in the spring.

(f) Annually, before storage season, check valves, gaskets, well guards, cover plates, flap gates on the outlet and other appurtenances (see EM 1110-2-1908, Part 1).

(g) Should be examined for stress cracks, bulges, shifts of alignment, excessive leakage, and debris cleaned from gutters and weirs.
control methods. The type of maintenance to be conducted on seepage control facilities is given in table 14-1.

14-4. Rehabilitation. Inspection and maintenance of seepage control facilities may indicate the need for rehabilitation. The type of rehabilitation to be conducted on various seepage control facilities is shown in table 14-1. If the seepage control facility cannot be satisfactorily rehabilitated, remedial seepage control facilities should be installed (see Chapter 12). The majority of rehabilitation of seepage control facilities is in connection with relief wells. Often a relief well does not flow except during high reservoir levels. The water in the well becomes stagnant, various chemicals precipitate, algae grows, and the efficiency of the well deteriorates. This will be manifested by a fall in relief well discharge accompanied by a rise in piezometric levels. Rehabilitation of a relief well is in essence a redevelopment of the well. In addition to mechanical methods such as water jetting, surging, compressed air, and pumping, certain chemicals, detergents, and water softeners can be used in the rehabilitation process. Chemical tests on water samples from the wells will indicate if and what chemicals are applicable. An examination of the well screen with a borehole TV camera is recommended, turbidity of the water permitting, to determine the degree of deterioration and/or clogging of the screen. For well screens constructed of metal (no wooden screens) that will not be damaged by acid, incrustation from calcium carbonate that has cemented gravel filter particles can be removed by treatment with hydrochloric acid. For well screens constructed of metal, treatment with chlorine can remove bacterial growths of slimes. Iron oxides may be removed by treatment with polyphosphate and surging the well. Calcium hypochlorite can be used with polyphosphates to kill iron bacteria (Johnson Division, Universal Oil Products Co. 1972).
APPENDIX A
REFERENCES

Government Publications
1. TM 5-818-5.
2. TM 5-545.
3. ER 1110-2-100.
5. EM 1110-2-1901.
6. EM 1110-2-1902.
7. EM 1110-2-1905.
8. EM 1110-2-1906.
10. EM 1110-2-1911.
11. EM 1110-2-1913.
12. EM 1110-2-2300.
13. EM 1110-2-2400.
15. EM 1110-2-3501.
16. EM 1110-2-3506.
17. EM 1110-2-4000.
18. Albritton, J., Jackson, L., and Bangert, R., "Foundation Grouting Practices at Corps of Engineers Dams," TR GL-84-13, 1984, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.
Banks, D. C. *Three-Dimensional Electrical Analogy Seepage Model Studies; Appendix B: Flow to a Single Well Centered Inside a Circular Source, Series H,* Technical Report No. 3-619, Mar 1965, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


Cobb, J. E., *Seepage Analysis Report - Stage 3, Specifications, Calamus Dam, Nebraska,* Technical Memorandum No. V-222-B-1g, Jan 1984, Bureau of Reclamation, P. O. Box 25007, Denver, CO 80225.


Cooper, S. S., and Bieganousky, W. A., *Geophysical Survey of Cavernous Areas, Patoka, Indiana,* Miscellaneous Paper S-78-1, Jan 1978, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.

Cooper, S. S., Koester, J. P., and Franklin, A. G., *Geophysical Investigation at Gathwright Dam,* Miscellaneous Paper GL-82-2, Mar 1982, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


35. Duncan, J. M., "Seepage Analysis for Columbia Lock and Dam," Miscellaneous Paper No. 3-503, Jun 1962, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


37. Edris, E. V., Jr., and Vanadit-Ellis, W., "Geotechnical Computer Program Survey," Miscellaneous Paper GL-82-1, Mar 1982, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


39. Franke, P., "The Concrete-Pile Cutoff Wall According to the I.C.O.S. - Vedner Patent," Translation No. 54-6, Sep 1954, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


42. Hall, R. L., Tracy, F. T., and Radhakrishnan, N. "Documentation of Two- and Three-Dimensional Seepage Problems Using the Finite Element Method," Miscellaneous Paper No. K-75-6, Jun 1975, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.

43. Hallford, C. R., "Fluid Trench Construction, Concrete Diaphragm Wall Construction," Lecture for Construction of Earth and Rockfill Dams Course, Mar 1983, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.

44. Hanson, J. E., "Impervious Cutoff by Means of Slurry Trench Construction Mill Site Dam and Reservoir, Ferron Creek, Ferron Utah," 1972, U. S. Department of Agriculture, Soil Conservation Service, P. O. Box 11350, Salt Lake City, UT 84147.


47. Holland, T. C., and Turner, J. R., "Construction of Tremie Concrete Cut-off Wall, Wolf Creek Dam, Kentucky," Miscellaneous Paper SL-80-10, Sep 1980, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


51. Leach, R. E., "Waterbury Dam Seepage Study," draft report prepared for U. S. Army Engineer District, New York, 1982, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.

53. Mansur, C. I., and Perret, W. R., "Efficacy of Partial Cutoffs for Controlling Underseepage Beneath Levees," Technical Memorandum No. 3-267, Jan 1949, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


56. McAnear, C. L., and Trahan, C. C., "Three-Dimensional Seepage Model Study, Oakley Dam, Sangamon River, Illinois," Miscellaneous Paper S-72-3, Jan 1972, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


58. Montgomery, R. L., "Investigation of Relief Wells, Mississippi River Levees, Alton to Gale, Illinois," Miscellaneous Paper S-72-21, Jun 1972, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.

63. Perry, E. B., "Laboratory Tests on Granular Filters for Embankment Dams," Draft Technical Report, 1986, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


65. Perry, E. B., "Susceptibility of Dispersive Clay at Grenada Dam, Mississippi, to Piping and Rainfall Erosion," Technical Report No. GL-79-14, Sep 1979, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.

66. Radhakrishnan, N., "Flow Nets and Computer Solutions to Seepage Problems," Seminar on Construction Dewatering and Pressure Relief, Jan-Feb 1978, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


74. Tracy, F. T., "An Interactive Graphics Postprocessor for Finite Element Method Results," Miscellaneous Paper K-77-4, Aug 1977b, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.

75. Tracy, F. T., "A Plane and Axisymmetric Finite Element Program for Steady-State and Transient Seepage Problems," Miscellaneous Paper K-73-4, May 1973a, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.

76. Tracy, F. T., "A Three-Dimensional Finite Element Program for Steady-State and Transient Seepage Problems," Miscellaneous Paper K-73-3, May 1973b, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


78. U. S. Army Engineer District, Baltimore, "Susquehanna River Basin, Kettle Creek, Pennsylvania, Kettle Creek Reservoir, Embankments," Design Memorandum No. 7, Oct 1958, P. O. Box 1715, Baltimore, MD 21203.


80. U. S. Army Engineer District, Ft. Worth, "Brazos River Basin, Aquilla Creek, Texas, Aquilla Lake, Embankment, Spillway, and Outlet Works," Design Memorandum No. 7, May 1976a, P. O. Box 17300, Ft. Worth, TX 76102.

81. U. S. Army Engineer District, Ft. Worth, "Trinity River Basin, Elm Fork, Trinity River, Aubrey Lake, Texas, Embankment and Spillway (Revised)," Design Memorandum No. 5, Jun 1976b, P. O. Box 17300, Ft. Worth, TX 76102.

82. U. S. Army Engineer District, Ft. Worth, "Trinity River Basin, Mountain Creek, Lakeview Lake, Texas, Embankment and Spillway," Design Memorandum No. 9, Apr 1980, P. O. Box 17300, Ft. Worth, TX 76102.

83. U. S. Army Engineer District, Galveston, "Buffalo Bayou and Tributaries, Addicks and Barker Dams, Texas, Emergency Seepage Control, Construction Completion Report," Jan 1983, P. O. Box 1229, Galveston, TX 77553.

84. U. S. Army Engineer District, Galveston, "Buffalo Bayou and Tributaries, Addicks Dam, Texas," Inspection Report No. 3, 1977a, P. O. Box 1229, Galveston, TX 77553.
85. U. S. Army Engineer District, Galveston, "Buffalo Bayou and Tributaries, Texas, Addicks Dam, Letter Report for Emergency Seepage Control," May 1977b, P. O. Box 1229, Galveston, TX 77553.

86. U. S. Army Engineer District, Galveston, "Buffalo Bayou and Tributaries, Texas, Major Rehabilitation, Addicks and Barker Dams," Reconnaissance Report, 1977c, P. O. Box 1229, Galveston, TX 77553.


90. U. S. Army Engineer District, Kansas City, "Hillsdale Lake, Osage River Basin, Big Bull Creek, Kansas," Plans for Hillsdale Dam, Apr 1978, 601 E. 12th Street, Kansas, City, MO 64106.


94. U. S. Army Engineer District, Louisville, "Taylorsville Lake, Ohio River Basin, Salt River, Kentucky, Dam and Spillway," Design Memorandum No. 6, Jan 1974, P. O. Box 59, Louisville, KY 60201.


96. U. S. Army Engineer District, Mobile, "Carters Dam, Coosawattee River, Georgia, Embankment Criteria and Performance Report," Mar 1976, P. O. Box 2288, Mobile, AL 36628.

97. U. S. Army Engineer District, Mobile, "Okatibbee Dam, Okatibbee Creek, Mississippi, Main Dam and Spillway," Design Memorandum No. 8, Apr 1965, P. O. Box 2288, Mobile AL 36628.
98. U. S. Army Engineer District, Nashville, "Wolf Creek Dam, Cumberland River, Kentucky, Concrete Diaphragm Walls, Plans and Specifications, Phase 1," Apr 1975, P. O. Box 1070, Nashville, TN 37202.


100. U. S. Army Engineer District, Omaha, "Missouri River, Oahe Reservoir, South Dakota, Relief Wells," Design Memorandum No. MO-162, Nov 1961, 215 N. 17th Street, Omaha, NE 68102.


111. U. S. Army Engineer District, St. Louis, "Clarence Cannon Dam and Reservoir, Upper Mississippi River Basin, Salt River, Missouri, Embankment Design, Main Dam," Design Memorandum No. 12, Nov 1969, 210 Tucker Boulevard, St. Louis, MO 63101.


113. U. S. Army Engineer District, Tulsa, "Big Hill Lake, Big Hill Creek, Kansas, Embankment, Outlet Works, and Spillway," Design Memorandum No. 6, Oct 1975, P. O. Box 61, Tulsa, OK 74121-0061.

114. U. S. Army Engineer District, Tulsa, "El Dorado Lake, Walnut River, Kansas, Embankment and Spillway," Design Memorandum No. 6, Jul 1974, P. O. Box 61, Tulsa, OK 74121-0061.

115. U. S. Army Engineer District, Vicksburg, "Operation and Maintenance Manual, Grenada Dam, Yalobusha River, Mississippi," 1977, P. O. Box 60, Vicksburg MS 39180-0631.


118. U. S. Army Engineer Division, Ohio River, "Conference on Control of Underseepage," Apr 1945, P. O. Box 1159, Cincinnati, OH 45201.

119. U. S. Army Engineer Waterways Experiment Station, "Analysis of Piezometer and Relief Well Data, Arkabutla Dam," Technical Report No. 3-479, Jun 1958, P. O. Box 631, Vicksburg/MS 39180-0631.

120. U. S. Army Engineer Waterways Experiment Station, "Investigation of Underseepage and Its Control, Lower Mississippi River Levees," 2 Vols, Technical Memorandum No. 3-424, Oct 1956a, P. O. Box 631, Vicksburg, MS 39180-0631.

121. U. S. Army Engineer Waterways Experiment Station, "Review of Soils Design, Construction, and Prototype Analysis, Blakely Mountain Dam, Arkansas," Technical Report No. 3-439, Oct 1956b, P. O. Box 631, Vicksburg, MS 39180-0631.

122. U. S. Army Engineer Waterways Experiment Station, "The Unified Soil Classification System," Technical Memorandum No. 3-357, Apr 1960, P. O. Box 631, Vicksburg, MS 39180-0631.


Non-Government Publications


165. Denny, K. J., Grain Size Distribution and Its Effect on the Permeability of Unconsolidated Sands, M.S. Thesis, 1965, University of Texas, Austin, Tex. Available from: Technical Information Center, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


171. Fenn, C. D., The Effects of Montmorillonite and Kaolinite Dispersions on the Permeability of a Porous Media, M.S. Thesis, 1966, Mississippi State University, State College, Miss. Available from: Technical Information Center, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.

172. Fetter, C. W., Jr. Applied Hydrology, Charles E. Merrill, Columbus, Ohio, 1980. Available from: Technical Information Center, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


174. Forcheimer, P. H., Hydraulik, Teubner, Berlin, 1914. Available from: Technical Information Center, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


177. Gebhart, L. R. "Foundation Seepage Control Options for Existing Dams," Inspection, Maintenance and Rehabilitation of Old Dams, American Society of Civil Engineers, New York, 1974, pp 660-676. Available from: Technical Information Center, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


from: Technical Information Center, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


Center, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


Information Center, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


Information Center, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


244. Radukic, V., "Observed Behavior of Embankment Dams at Some Typical Interfaces," *Proceedings, Thirteenth International Congress on Large Dams*, India, Vol 1, 1979, pp 1005-1922. Available from: Technical Information Center, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


A-25


274. Terzaghi, K., Theoretical Soil Mechanics, Wiley, New York, 1943. Available from: Technical Information Center, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


278. Todd, D. D., and Bear, J., "River Seepage Investigation," Water Resources Center Contribution No. 20, Sep 1959, Hydraulic Laboratory, University of California, Berkeley. Available from: Technical Information Center, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


280. Turkish National Committee, "Investigation of Leaks at Keban Dam," Transactions of the Twelfth International Congress on Large Dams, Vol II, 1976, Mexico City, pp 459-483. Available from: Technical Information Center, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.

Available from: Technical Information Center, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631.


B-1. Introduction. As previously mentioned in Chapter 4, various methods are available, in addition to flow nets, for solving idealization of seepage problems. As shown in Figure 4-3, these methods include electrical analogy, hydraulic or sand tank models, viscous flow models, method of fragments, finite difference method, and finite element method. Prior to conducting an analysis, the problem to be studied must be defined in terms of:

a. Aquifer and embankment dimensions.

b. Coefficients of permeability of the embankment and foundation soils.

c. Horizontal to vertical permeability ratios.

d. Boundary conditions (impermeable and symmetrical).

e. Exits and entrances (fixed potential areas).

f. Head versus time relationships for unsteady flow.

Sensitivity studies may be run to establish the effect of parameters not known accurately.

B-2. Electrical Analogy.

a. General. Processes which involve movement of current due to differences in energy potential operate on the same principles as movement of confined ground water as shown in Table B-1. Therefore, to obtain the pattern of equipotential lines or flow lines (see Figure 4-4), the flow domain is transferred by an electrical conductor of similar geometric form as first proposed by Pavlovsky in 1918 (Harr 1962). Electrical analogies may involve two-dimensional conducting paper models or three-dimensional tanks containing aqueous solution.

b. Two-Dimensional Models. When field conditions can be approximated by a two-dimensional plan or section, teledeltos conducting paper models may be used to obtain a flow net. Two-dimensional teledeltos models are simple to use and can accommodate various geometries. However, it is difficult to simulate varying permeabilities and they are generally restricted to steady state confined aquifers (Bear 1972, Boer and Molen 1972).

c. Three-Dimensional Models. The use of electrical analogy models is described by various authors (Zangar 1953, Todd and Bear 1959, and Duncan 1963). The three-dimensional electrical analogy model at the U. S. Army Engineer Waterways Experiment Station (WES) (see Figure B-1) is a plexiglass tank filled with dilute copper sulfate solution and having a calibrated elevated carrier assembly for the accurate placement of a point electrode probe.
Table B-1. Analogy Between Darcy's Law and Ohm's Law

<table>
<thead>
<tr>
<th>Darcy's Law</th>
<th>Ohm's Law</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q = \frac{KAH}{L} )</td>
<td>( I = \frac{K'A'V}{L'} )</td>
</tr>
<tr>
<td>Q = rate of flow or water</td>
<td>I = current (rate of flow of electricity)</td>
</tr>
<tr>
<td>K = coefficient of permeability</td>
<td>K' = conductivity coefficient</td>
</tr>
<tr>
<td>A = cross-sectional area</td>
<td>A' = cross-sectional area</td>
</tr>
<tr>
<td>H = head producing flow</td>
<td>V = voltage producing current</td>
</tr>
<tr>
<td>L = length of path of percolation</td>
<td>L' = length of path of current</td>
</tr>
</tbody>
</table>

[a] From Bureau of Reclamation.

anywhere in the fluid. Extensive use of the WES model has been made to:

(1) Determine uplift values and seepage quantities for use in the design of Columbia Lock and Dam, Louisiana (Duncan 1962).

(2) Determine the uplift values and seepage quantities for fully and partially penetrating well arrays from line and circular sources (Duncan 1963, Banks 1963, and Banks 1965).

(3) Determine uplift pressures beneath the spillway, piezometric heads at the downstream toe of the dam, and total seepage quantities for use in the design of Oakley Dam, Illinois (McAnear and Trahan 1972).

B-3. Sand Tank Model. The sand tank model (hydraulic model), as shown in figure B-2, consists of a rigid, watertight container with a transparent front, filled with sand, deaired water, and measuring devices. The geometry of the sand tank corresponds to that of the prototype. The sand may be placed under water to provide a homogeneous condition, or layers of different sand sizes may be used to study anisotropy. If the flow is unconfined and the same material is used for model and prototype, the capillary rise must be compensated for in the model. When a steady-state flow is reached, dye can be introduced at various points along the upstream boundary close to the front wall to form traces of the streamlines. Piezometers are used to measure the pressure heads at various locations (Bear 1972 and Harr 1962). A sand tank model was employed to investigate the effect of length of horizontal drain on the through seepage flow nets and quantities for a homogeneous and isotropic sand embankment.

For prolonged tests, disinfectants such as Formol should be added to the water to prevent bacterial growth that causes clogging (Bear 1972).
Figure B-1. Three-dimensional electrical analogy apparatus (after Duncan)
PARTIALLY PENETRATING WELL
WITH A CIRCULAR SOURCE

SEEPAGE FROM CANALS

TRACER DYE

SEEPAGE IN EMBANKMENTS

Figure B-2. Hydraulic or sand tank model
(prepared by WES)
Sand tank models are also used extensively in petroleum engineering, ground-water quality, and pollution research (Bear 1972 and Prickett 1979).

**B-4. Viscous Flow Models.** The viscous flow model, also called the Hele-Shaw or parallel plate model, is based on the similarity between the differential equations governing saturated flow in a porous medium and those describing the flow of a viscous liquid in the narrow space between two parallel plates. The viscous flow model contains the shape of the structure to be investigated and once a steady-state flow is obtained, colored dyes can be injected along the upstream edge and patterns of streamlines can be observed. A camera (movie or still) is normally used to record the results of experiments. Inhomogeneous hydraulic conductivity, such as would exist in a zoned earth dam, can be simulated by varying the width of the interspace between the parallel plates, as shown in figure B-3. The viscous flow model experiments should be conducted in a temperature-controlled room because viscosity plays an important role in analog scaling. If this is not feasible, the temperature should be measured at all inflow and outflow points during the test and scales must be recomputed according to the varying average temperature of the liquid in the model (Bear 1972 and Harr 1962). A viscous flow model was constructed at WES to simulate seepage conditions induced in streambanks by sudden drawdown of the river level. The results from the model study compared favorably with field observations, finite difference, and finite element methods (Desai 1970 and Desai 1973).

**B-5. Method of Fragments.**

a. **General.** The method of fragments is an approximate analytical method for the computation of flows and pressure heads for any ground-water system. The underlying assumption of this procedure developed by Pavlovsky in 1935 (Pavlovsky 1956 and Harr 1962) is that equipotential lines at various critical locations in the flow region can be approximated by straight vertical lines. These equipotential lines divide the flow region into parts or fragments. Other assumptions inherent in the method of fragments procedure are (a) Darcy's law is valid, (b) steady-state flow exists, and (c) the soil medium is approximated as a single homogeneous and isotropic layer or at series of such layers. The transformation of anisotropic soil to an equivalent isotropic soil is described in Section 4.7 of this manual.

b. **Basic Concepts.** The quantity of flow through a single fragment is computed as:

\[
q = \frac{k h_i}{\phi_i} \quad (B-1)
\]

where

- \( k \) = coefficient of permeability
- \( h_i \) = head loss through the fragment
- \( \phi_i \) = dimensionless form factor, \( = \frac{N_e}{N_f} \)
Figure B-3. Viscous flow model (courtesy of Bear)
Because the fragment boundaries consist of equipotential lines, the flow through each fragment must be equal to the total flow through the system. Thus

\[ Q = \frac{kh_1}{\phi_1} = \frac{kh_2}{\phi_2} = \cdots = \frac{kh_n}{\phi_n} \quad (B-2) \]

Since summation of the head loss in each fragment is equal to the total head loss, the total quantity of flow can be expressed as

\[ Q = \frac{kh}{\sum_{i=1}^{n} \phi_i} \quad (B-3) \]

where \( h \) is the total head loss through the section. Along the same line, the head loss in each fragment can be calculated from

\[ h_i = \frac{h\phi_i}{\sum_{i=1}^{n} \phi_i} \quad (B-4) \]

The head loss along any impermeable boundary of a fragment is assumed to change linearly. Thus the head loss within fragment \( i \) up to point \( A \) is equal to the head loss in the fragment times the ratio of the length of the boundary to point \( A \) to the total length of boundary. The basic concept of the method of fragment procedure is to break the flow region into parts for which the form factor is shown in figure B-4 (Harr 1977). This manual will describe how to calculate the factors for each type of fragment (Harr 1962 and Harr 1977).

c. Fragment Types. There are currently nine different fragment types. Of these, the first six are for confined flow while the last three are for unconfined flow.

(1) Type I. This fragment type represents a region of parallel horizontal flow between impervious boundaries. For this internal type fragment, shown in figure B-5a, the flow per unit width is equal to

\[ Q = \frac{kh_1 a}{L} \quad (B-5) \]
Thus from Equation B-1, the form factor is

\[
\phi = \frac{L}{a} \quad \text{(B-6)}
\]

An elemental Type I section shown in figure B-5b illustrates that

\[
d\phi = \frac{dx}{y} \quad \text{(B-7)}
\]

This elemental section will be used to derive the form factors for fragment types IV, V, and VI.

(2) Type II. This fragment type represents a vertical impervious boundary embedded a length \( S \) into a pervious layer of thickness \( T \). This fragment can represent either an entrance condition (figure B-6a) or an exit condition (figure B-6b). The form factor is obtained from the plot in figure B-4 where the scale of \( \phi \) is given as one-half the reciprocal of \( Q/kh \) or

\[
\phi = \frac{1}{2} \left( \frac{kh}{Q} \right) \quad \text{(B-8)}
\]

The form factor could also be expressed as the ratio of the elliptic integral of the first kind with modulus \( m \) over the elliptic integral of the complementary modulus, \( m' \). For this fragment type, the modulus value is a function of the ratio \( S/T \). The graph in figure B-7 was obtained by solving the elliptic integrals for various combinations of \( S/T \). For the type II fragments, the ratio of \( b/T \) equals 0.

(3) Type III. This type of fragment represents an impervious layer of length \( b \), a vertical boundary of depth \( S \), in a pervious layer of thickness \( T \). Either of the sections shown in figure B-8 can represent this fragment type. The form factor is obtained directly from figure B-7 with \( b/T \) other than zero. For this case, the elliptic integral modulus is a function of both \( b/T \) and \( S/T \).

(4) Type IV. This type is an internal fragment with boundary length \( b \), embedment length \( S \), in a pervious layer of thickness \( T \). Figure B-9a illustrates the two possible configurations. Pavlovsky divided the flow region into active and passive parts based on the results of electrical analogue tests as shown in figure B-9b by line \( AB \). An angle of 45 deg was assumed for the line dividing the two parts of the fragment. This resulted in two cases, depending on the relation between \( b \) and \( S \). For the case where \( b < S \), the
<table>
<thead>
<tr>
<th>Fragment type</th>
<th>Illustration</th>
<th>Form factor, $\Phi$ (h is head loss through fragment)</th>
<th>Fragment type</th>
<th>Illustration</th>
<th>Form factor, $\Phi$ (h is head loss through fragment)</th>
</tr>
</thead>
</table>
| I             | ![Illustration](image1.png) | $\Phi = \frac{L}{a}$ | V             | ![Illustration](image2.png) | $L \leq 2s$: $\Phi = 2 \ln \left(1 + \frac{L}{2a}\right)$  
$L \geq 2s$: $\Phi = 2 \ln \left(1 + \frac{s}{a}\right) + \frac{L - 2s}{T}$ |
| II            | ![Illustration](image3.png) | $\Phi = \frac{1}{2} \left(\frac{k}{Q}\right)$, Fig. 5-13 | VI            | ![Illustration](image4.png) | $L \geq s' + s'':$  
$\Phi = \ln \left[\left(1 + \frac{b'}{a''}\right) \left(1 + \frac{b''}{a'}\right)\right]$  
+$ \frac{L - (s' + s'')}{T}$  
$L \leq s' + s'':$  
$\Phi = \ln \left[\left(1 + \frac{b'}{a''}\right) \left(1 + \frac{b''}{a'}\right)\right]$  
where $b' = \frac{L + (s' - s'')}{2}$  
$b'' = \frac{L - (s' - s'')}{2}$ |
| III           | ![Illustration](image5.png) | $\Phi = \frac{1}{2} \left(\frac{k}{Q}\right)$, Fig. 5-13 | VII           | Streamline | $\Phi = \frac{2L}{h_1 + h_2}$  
$Q = k \frac{h_1 - h_2}{2L}$ |
| IV            | ![Illustration](image6.png) | $b \leq s$: $\Phi = \ln \left(1 + \frac{b}{a}\right)$  
$b \geq s$: $\Phi = \ln \left(1 + \frac{s}{a}\right) + \frac{b - s}{T}$ | VIII          | ![Illustration](image7.png) | $Q = k \frac{h_1 - h}{\cot \alpha} \ln \frac{k_2 - h}{h_2 - h}$ |
| IX            | ![Illustration](image8.png) | $Q = k \frac{a_2}{\cot \beta} \left(1 + \ln \frac{a_2 + h_1}{a_2}\right)$ |

Figure B-4. Summary of fragment types and form factors (courtesy of McGraw-Hill Book Company)
Figure B-5. Type I fragment (courtesy of McGraw-Hill Book Company\textsuperscript{181}).

Figure B-6. Type II fragment (courtesy of McGraw-Hill Book Company\textsuperscript{181}).
Figure B-7. Quantity of discharge for symmetrically placed pilings (courtesy of McGraw-Hill Book Company)

\[
\frac{Q}{kh} = \frac{1}{2\Phi}
\]

If \( m = \cos \frac{\pi s}{2T} \sqrt{\tanh^2 \frac{\pi b}{2T} + \tan^2 \frac{\pi s}{2T}} \)

for \( m \leq 0.3 \), \( \frac{Q}{kh} = \frac{1}{\pi} \ln \frac{4}{m} \)

for \( m^2 \geq 0.9 \), \( \frac{Q}{kh} = \frac{-\pi}{2 \ln \left( \frac{1}{1-m^2} \right)} \)
active zone is composed of elements of type I fragments of width $dx$ illustrated in figure B-9c. The form factor is the integral of $dx$ over $y$ from 0 to $b$ which results in a form factor of

$$
\phi = \ln \left( 1 + \frac{b}{a} \right)
$$

(B-9)

If $b > S$, then the fragment can be divided into two fragments as shown in figure B-9e. The first is a type IV with $b > S$ and the second is a type I fragment with $L$ equal to $b - S$. Thus the form factor is the sum of the form factors which would be

$$
\phi = \ln \left( 1 + \frac{S}{a} \right) + \frac{b - S}{T}
$$

(B-10)

(5) Type V. This fragment type has two vertical boundaries of equal embedment $S$ in a pervious layer of thickness $T$. As shown in figure B-10, the form factor for this fragment is twice that for the type IV fragment.
Figure B-9. Type IV fragment (courtesy of McGraw-Hill Book Company)
Since there were two cases of type IV fragment, there are two cases for the type V fragment. The two cases are for $L \leq 2s$ and $L \geq 2S$. For the first case, the form factor is

$$\Phi = 2\ln\left(1 + \frac{L}{2a}\right)$$  \hspace{1cm} (B-11)

For the second case which consists of a type I fragment within two type IV fragments, the form factor is

$$\Phi = 2\ln\left(1 + \frac{S}{a}\right) + \frac{L - 2S}{T}$$  \hspace{1cm} (B-12)

(6) Type VI. This fragment type, illustrated in figure B-11, is the same as the type V fragment except that the embedment lengths are different. Using the same approximations as in fragment type IV, there are two cases for the form factor. For the first case where $L > (S' + S'')$, the form factor is

$$\left[\left(1 + \frac{S'}{a}\right)\left(1 + \frac{S''}{a}\right)\right] + \frac{L - (S' + S'')}{T}$$  \hspace{1cm} (B-13)
For the second case where $L < (S' + S'')$, the form factor is

$$\phi = \ln \left( \frac{L}{a'} \right)$$

(B-14)

where

$$b' = \frac{L + (S' - S'')}{2}$$
$$b'' = \frac{L - (S' - S'')}{2}$$

(7) Type VII. This fragment represents the condition of unconfined flow. This flow is characterized by having one boundary of the flow domain as a free surface (line AB in figure B-12). This free surface separates the saturated region from that region where no flow occurs. From Darcy's law and Dupuit's assumptions, the hydraulic gradient is $(h_1 - h_2)/L$ and the cross-sectional area is $(h_1 + h_2)/2$, thus the flow is

$$Q = k \frac{h_1^2 - h_2^2}{2L}$$

(B-15)
Figure B-12. Type VII fragment (courtesy of McGraw-Hill Book Company)

From this, the form factor is

\[ \phi = \frac{2L}{h_1 + h_2} \]  

(B-16)

(8) Type VIII. This fragment type represents an upstream slope entrance condition on an earth dam of height \( h_d \) and is illustrated in figure B-13. It was assumed that the curve streamlines (cd) could be approximated by horizontal flow channels of length \( e_d \) (Pavlovsky 1956 and Harr 1977). With this assumption, the hydraulic gradient in each channel is

\[ i = \frac{d(h_1 - h)}{dy} = \frac{a_1}{\cot \alpha (h_d - y)} \]  

(B-17)
Integrating the ratio dy/(h_d - y) from 0 to h generates the expression for the quantity of flow as

\[
Q = k \left( \frac{h_1 - h}{\cot \alpha} \ln \frac{h_d}{h_d - h} \right)
\]

(9) Type IX. This fragment type, shown in figure B-14, represents the exit condition where the surface of seepage exists. The surface of seepage (DE) is not an equipotential line or a streamline. Pavlovsky assumed that the flow is horizontal. For the portion of the slope between D and E, the flow is the coefficient of permeability times the integral of dy over cot \( \beta \). The flow for E to F is the permeability times the integral of \( a_2 dy \) over the cot \( \beta \) \( (a_2 + h_2 - y) \). When the integration is performed, the expression for the flow is

\[
Q = \frac{ka_2}{\cot \beta} \left( 1 + \epsilon \frac{a_2 + h_2}{a_2} \right)
\]
Figure B-14. Type IX fragment (courtesy of McGraw-Hill Book Company)

d. Exit Gradient. The method of fragments procedure can be used to determine the exit gradient discussed in paragraph 4.9 of this manual. For this procedure, the last fragment (downstream) needs to be either a type II or a type III fragment. The exit gradient is defined as (Harr 1962)

\[ I_E = \frac{h_m \pi}{2KT_m} \]  

(B-20)

where

- \( h_m \) = head loss in the last fragment
- \( K \) = complete elliptic integral of the first kind with modulus \( m \)
- \( T \) = depth of flow region

As defined before, the modulus \( m \) is a function of both \( b/T \) and \( S/T \) and is defined as

\[ m = \cos \frac{\pi S}{2T} \sqrt{\tanh^2 \frac{\pi b}{2T} + \tan^2 \frac{\pi S}{2T}} \]  

(B-21)
Instead of calculating the various values, for type II fragments figure B-15 can be used with \( S/T \) to obtain the fraction for \( I_n S/h_m \). By substituting the appropriate values, the exit gradient is calculated.

e. Example 1. The method of fragment procedure for confined flow will be illustrated in the following example obtained for John H. Overton Lock and Dam (U. S. Army Engineer District, St. Louis 1978). This problem will analyze the steady state flow conditions for a two-dimensional idealization of the lock structure. The quantity of flow and head along the bottom of the lock will be determined. For illustrative purposes, the exit gradient procedure will be included. The dimensions of the structure, shown in figure B-16, are those used in the analysis after the cross section has been transformed to account for soil anisotropy. The original analysis contained two soil layers, but for illustrative purposes the soil will be modeled as one layer. The first step is to determine the form factors for each region. The first region is a type II fragment with \( S = 19 \) ft and \( T = 89 \) ft. Using figure B-7 with \( S/T = 0.21 \), the fraction for \( Q/kh = 0.78 \), thus \( \phi_1 = 0.641 \). Region 2 is a type I fragment with \( L = 456 \) ft and \( a = 70 \) ft. From figure B-4, the form factor for the type I fragment is equal to \( L/a \), thus \( \phi_2 = 6.514 \). The third region is a type II fragment with \( S = 9 \) ft and \( T = 79 \) ft. Using figure B-7 with \( S/T = 0.114 \), the fraction for \( Q/kh = 1.01 \), thus \( \phi_3 = 0.495 \). The summation of the form factors is 7.650. The quantity of flow is calculated from equation B-3. Using transformed permeability of \( 400 \times 10^{-4} \) cm/sec and a total head of 18 ft, the quantity of flow is calculated to be 266.8 ft\(^3\)/day/foot of lock width. The head loss in each fragment is calculated from equation B-4. The following table lists the head loss for each fragment in this problem:

<table>
<thead>
<tr>
<th>Region</th>
<th>( \phi )</th>
<th>( h_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.641</td>
<td>1.51</td>
</tr>
<tr>
<td>2</td>
<td>6.514</td>
<td>15.33</td>
</tr>
<tr>
<td>3</td>
<td>0.495</td>
<td>1.16</td>
</tr>
<tr>
<td>( \Sigma = 7.650 )</td>
<td>( \Sigma = 18.00 )</td>
<td></td>
</tr>
</tbody>
</table>

The head along the bottom decreases from 16.49 ft at the upstream end to 1.16 at the downstream end. Using the assumption of a linear distribution of the head loss within a fragment, the head at any point along the bottom of the lock could be calculated as

\[
\text{head at pt } a = 16.49 \text{ ft} - \left( \frac{\text{distance to pt } A \text{ for upstream of lock}}{\text{total length of lock}} \right) 15.33 \text{ ft} \quad (B-22)
\]
Figure B-15. Relationship for determining the exit gradient as a function of the head loss in the fragment and geometry for a Type II fragment (courtesy of McGraw-Hill Book Company).
Figure B-16. Transformed section of John H. Overton Lock simplified to one soil layer (from U. S. Army Engineer District, St. Louis 1978)

The exit gradient is calculated for region 3 which is a type II fragment. Using $S/T = 0.114$ with figure B-15, the fraction for $I_E S/h_m$ is found to be 0.63. With a head loss of 1.16 ft in this fragment, the exit gradient is calculated to be 0.082.

f. Example 2. This example will illustrate the method of fragment procedure for unconfined flow problems. The example is obtained from John H. Overton Lock and Dam (U. S. Army Engineer District, St. Louis 1978). The problem is to locate the free surface in the closure dam and to determine the quantity of flow through the dam under steady state conditions. The dimensions of the structure shown in figure B-17 are after the material has been transformed to account for soil anisotropy. This sample problem assumes an impervious boundary at the base of the dam. To account for some flow under the dam, the impervious boundary could be lowered. By lowering the boundary to the lowest possible point, bounds for the problem would be established. There are three fragment types in this earthen embankment. Region 1 is a
Figure B-17. Transformed section of John H. Overton closure dam (from U. S. Army Engineer District, St. Louis\textsuperscript{112})

type VIII fragment while region 2 is a type VII fragment and region 3 is a type IX fragment. To calculate the flow through region 1, equation B-18 is used with $h_1 = 32$ ft; $h_d = 37$ ft, and $\alpha = 19.9$ deg (cot $\alpha = 2.76$).

Substituting into the equation produces

\[
\frac{Q}{k} = \frac{32 - h}{2.76} \ln \frac{37}{37 - h} \quad (B-23)
\]

For region 2, the quantity of flow is calculated from equation B-15.

Substituting into this equation produces

\[
\frac{Q}{k} = \frac{h^2 - (a_2 + 14 \text{ ft})^2}{2L} \quad (B-24)
\]

For region 3, equation B-19 defines the quantity of flow. By substituting cot $\beta = 2$ and $H_2 = 14$ ft produces

\[
\frac{Q}{k} = \frac{a_2}{2} \left(1 + \ln \frac{a_2 + 14}{a_2}\right) \quad (B-25)
\]

B-22
From the embankment geometry, \( L \) can be defined as

\[
L = b + \cot \beta \left[ h_d - (a_2 + h_2) \right]
\]  \hspace{1cm} (B-26)

By substituting into equation B-26, there are four equations with four unknowns, \( h \), \( a_2 \), \( Q/k \), and \( L \). There are several methods to solve these four equations (Harr 1977). For the case where \( h_2 = 0 \), a reduction of two equations and two unknowns occurs. For this example, equation B-23 will be combined with equation B-24 and equation B-26 will be substituted for \( L \). This produces

\[
\frac{32 - h}{2.76} \ln \left( \frac{37}{37 - h} \right) = \frac{h^2 - (a_2 + 14)^2}{2[133 + 2(37 - (a_2 + 14))]} \]  \hspace{1cm} (B-27)

Also equation B-23 can be combined with equation B-25, producing

\[
\frac{32 - h}{2.76} \ln \left( \frac{37}{37 - h} \right) = \frac{a_2}{2} \left( 1 + \ln \frac{a_2 + 14}{a_2} \right) \]  \hspace{1cm} (B-28)

Equations B-27 and B-28 have reduced the equations and unknowns by two. Thus with two equations and two unknowns, a trial and error graphical process can be used. The results of this process are shown in figure B-18 and indicate that \( h = 28.9 \) ft and \( a_2 = 0.9 \) ft. Substituting into equations B-23 and B-25 generates a \( Q/k \) value of 1.71 which results in an estimated flow of 242.4 \( \text{ft}^3/\text{day/ft of dam} \). Knowing \( h \) and \( a_2 \), the location of the phreatic surface can be estimated.

g. Flow in Layered Systems. One of the limitations of the method of fragments is that the flow layer is assumed to be homogeneous and isotropic. An approximate procedure to determine flow characteristics of a layered system was proposed by Polubarinova-Kochina (1941). Harr (1977) extended this method as follows. The coefficients of permeability for the two layers are related by a dimensionless parameter \( \varepsilon \) by the expression

\[
\tan \pi \varepsilon = \sqrt{\frac{k_2}{k_1}} \]  \hspace{1cm} (B-29)

where

\( k_1 = \) coefficient of permeability of the upper layer

\( k_2 = \) coefficient of permeability of the lower layer

B-23
\[
\frac{32 - h}{2.76} \ln \left( \frac{37}{37 - h} \right) = \frac{h^2 - (a_2 + 14)^2}{2(133 + 2[37 - (a_2 + 14)])}
\]

Equation B-27

<table>
<thead>
<tr>
<th>h</th>
<th>a_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>28.8</td>
<td>-0.4</td>
</tr>
<tr>
<td>29.0</td>
<td>2.1</td>
</tr>
<tr>
<td>30.0</td>
<td>8.6</td>
</tr>
<tr>
<td>31.0</td>
<td>13.6</td>
</tr>
</tbody>
</table>

\[
\frac{32 - h}{2.76} \ln \left( \frac{37}{37 - h} \right) = \frac{a_2^2}{2} \left( 1 + \ln \left( \frac{a_2 + 14}{a_2} \right) \right)
\]

Equation B-28

<table>
<thead>
<tr>
<th>h</th>
<th>a_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>29.9</td>
<td>0.6</td>
</tr>
<tr>
<td>27.8</td>
<td>1.2</td>
</tr>
<tr>
<td>25.6</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Figure B-18. Graphical solution of equations B-27 and B-28 (from U. S. Army Engineer District, St. Louis)
The ratio of permeabilities can vary from 0 to infinity. Over this range, \( \varepsilon \) ranges from 0 to 1/2. The basis of this method is to determine the flow and head losses for three certain special cases of \( \varepsilon \) and then interpolate between these values. The three special cases are as follows:

1. \( \varepsilon = 0 \). For \( \varepsilon \) to be equal to 0, \( k_2 \) must equal 0. Therefore the problem is reduced to a one-layer problem with a flow region thickness equal to the upper layer.

2. \( \varepsilon = 1/4 \). For \( \varepsilon \) to be equal to 1/4, \( k_2 \) must equal \( k_1 \). Therefore, the problem is reduced to a one-layer problem with a flow region the thickness of the upper and lower layers.

3. \( \varepsilon = 1/2 \). For \( \varepsilon \) to be equal to 1/2, \( k_2 \) must be infinite. This case represents the infinite flow where there is no resistance to flow in the lower layer. Since \( Q/k_1h = \infty \) the inverse of this ratio is equal to zero.

This procedure can be expanded to a three-layer system by the use of two \( \varepsilon \) values. The first value would be for the top two layers, while the second would be for the bottom two layers.

h. Example 3. This example will illustrate the method of fragment procedure for confined flow in a two-layer system. The example is obtained from John H. Overton Lock and Dam (U. S. Army Engineer District, St. Louis 1978). This problem will analyze the steady-state flow conditions for the dam and stilling basin. The quantity of flow and head along the bottom of the structure will be determined. For illustrative purposes, the exit gradient procedure will be included. The effect of various parameters like the length of sheetpile cutoff can be studied using this procedure. The dimensions of the structure, shown in figure B-19, are those used in the analysis after the cross section has been transformed to account for soil anisotropy. There are three fragments for this problem and three \( \varepsilon \) cases to be evaluated. For the first case, \( \varepsilon = 0 \), all the flow is assumed to occur in the clay layer. Region 1 is a type II fragment with \( S = 25 \) ft and \( T = 35 \) ft. Using figure B-7 with \( S/T = 0.71 \), the fraction for \( Q/k_1h = 0.36 \), thus \( \phi_1 = 1.38 \). The second region is a type V fragment with \( S = 13 \) ft, \( T = 23 \) ft, and \( L = 73.5 \) ft. Since \( L > 25 \), equation B-13 is used to calculate the form factor. For the above values, the form factor is 3.73. Region 3 is a type II fragment where \( S = 24 \) ft and \( T = 34 \) ft. The form factor, using figure B-7, is calculated to be 1.36. Using equation B-1, the ratio \( Q/k_1 \) is 2.78 and \( k_1/Q \) is 0.36.

For the second case, \( \varepsilon = 1/4 \), the flow is assumed to be equal in both layers. The form factors are recalculated using the same fragment types. The value of \( Q/k_1 \) is 6.50 which results in a \( k_1/Q \) value of 0.15. For the last case, \( \varepsilon = 1/2 \), all flow is assumed to be in the lower sand layer. Only vertical flow occurs in the top or clay layer. For this case, \( Q/k_1 \) is infinite which results in a \( k_1/Q \) of 0. A plot of \( k_1/Q \) versus \( \varepsilon \) is shown in figure B-20a. For this problem \( k_2 \) is 200 times \( k_1 \), therefore \( \varepsilon \) equals 0.48.
Figure B-19. Transformed section of John B. Overton Dam and stilling basin for one case of sheet pile lengths (from U. S. Army Engineer District, St. Louis) 

By interpolation for $\varepsilon = 0.48$, $k_i/Q$ equals 0.01 which results in a flow $Q$ of 56.7 ft$^3$/day/ft of dam. To determine the head along the bottom of the structure, the head at points A and B in figure B-19 must be determined. Using the procedure described in example 1, equation B-22, the following head loss and total head values are calculated.

<table>
<thead>
<tr>
<th></th>
<th>Point A</th>
<th>Point B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Head Loss</td>
<td>Total Head</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>ft</td>
<td>ft</td>
</tr>
<tr>
<td>0</td>
<td>5.2</td>
<td>52.8</td>
</tr>
<tr>
<td>1/4</td>
<td>6.2</td>
<td>51.9</td>
</tr>
</tbody>
</table>

For the case where $\varepsilon = 1/2$, the head anywhere along the bottom of the structure is equal to half the total head loss, or for this case 9 ft. Thus the total head on points A and B is equal to 49 ft. Figure B-20b is the plot of the total head versus $\varepsilon$ and shows, for an $\varepsilon$ of 0.48, the total head at point A is 49.4 ft while the total head at point B is 48.5 ft. The exit gradient for each $\varepsilon$ case is calculated by the procedure described in example 1. For the case of $\varepsilon = 0$, the fraction $I_p S/h_m$ is 0.55 which with $S = 24$ ft
Figure B-20. $\epsilon$ value plots for John H. Overton Dam and stilling basin (from U. S. Army Engineer District, St. Louis\textsuperscript{112})
and \( h_m = 3.8 \) ft produces \( I_E = 0.087 \). For the \( \epsilon = 1/4 \) case, the fraction 
\( I_E S/h_m \) is 0.615 which with \( S = 24 \) ft and \( h_m = 5.0 \) ft produces 
\( I_E = 0.128 \). For the case where \( \epsilon = 1/2 \), the head loss is the total head and 
the distance is the thickness of the top layer. Using the equation 

\[
I_E = \frac{h_m}{2d_1}
\]

the exit gradient is 0.265. The exit gradient versus \( \epsilon \) plot is shown in 
figure B-20c. For an \( \epsilon \) value of 0.48, the exit gradient is 0.245.

i. Uses and Limitations. The method of fragment procedure should be 
used as a design tool where various factors are changed to evaluate their 
effect or as an analytical tool when quick approximate results are needed. 
When numerous factors are varied, the construction of flow nets becomes very 
tedious and time consuming. The method of fragment procedure will generate 
reasonable results for problems where the assumptions are not greatly violated. 
There are several points the user needs to be aware of when using this proce-
dure. The flow region must be generalized so that it consists of horizontal 
and vertical boundaries. The procedure models the actual flow paths within 
the flow region, thus if there is any doubt as to the direction, a rough flow 
et should be drawn. This becomes important when a small portion of a struc-
ture is modeled with several fragments because the flow could be modeled in 
unnatural paths. The accuracy of the results is dependent upon how well the 
fragment boundary actually represents vertical equipotential lines. The 
greater the deviation, the greater the degree of error. However, for many 
practical problems reasonable results are generated. Comparison of the method 
of fragment results with finite element solutions for a one-layer system 
showed that the quantity of flow values for the fragment procedure are within 
8 percent of the finite element results, while the uplift values are within 
38 percent of the finite element results. The Computer-Aided Structural Engi-
eering (CASE) project has developed a computer program for the method of 
fragments procedure with a user manual describing the program (Pace et al. 
1984).

B-6. Finite Difference Method.

a. Method of Solution. As previously mentioned in Chapter 4, the 
finite difference method solves the Laplace equations by approximating them 
with a set of linear algebraic expressions. The approximation is mathematical 
rather than physical. The early methods of solving finite difference expres-
sions for Laplace's equation were based upon hand calculations by the relaxa-
tion method\(^{(1)}\). However, more recently a wide range of finite difference 
solutions suited to the digital computer have been developed. A description of 

\(^{(1)}\) For example, see Appendix A of EM 1110-2-2501.
available methods used to solve finite difference problems including example applications, case studies, and computer program listings is available (Rushton and Redshaw 1979).

b. **Advantages.** Use of iterative techniques such as successive over-relaxation which converge to the correct solution allows solution of unconfined and transient flow problems. For simple problems, the finite difference method is usually more economical than the finite element method (Rushton and Redshaw 1979).

c. **Disadvantages.** The finite difference method is not suited to complex geometry, including sloping layers and pockets of materials of varying permeability. Irregular grids are difficult to input. Therefore, zones where seepage gradients or velocities are high cannot be accurately modeled (Rushton and Redshaw 1979).

d. **Applications.** The finite difference method was used at WES to simulate seepage conditions in streambanks induced by sudden drawdown of the river level. As mentioned previously, this study included a viscous flow model, field observations, and application of the finite element and finite difference methods. The results of the study indicated that the finite difference method provided satisfactory and economical solutions for transient unconfined fluid flow in porous, anisotropic, and nonhomogeneous media (Desai 1970 and Desai 1973). The finite difference method was used to predict the location of the phreatic surface within a zoned embankment with arbitrary fluctuations of the reservoir (Dvinoff 1970). Generalized digital computer programs have been developed which use the finite difference method to simulate one-, two-, and three-dimensional nonsteady flow problems in heterogeneous aquifers under water table and artesian conditions (Prickett and Lonnquist 1971 and Desai 1977). The finite difference method has been used to predict unsteady flow in gravity wells. Good agreement was found between computed results and laboratory test results obtained using a sand tank model (Desai 1977).

**B-7. Finite Element Method.**

a. **Method of Solution.** As previously mentioned in Chapter 4, the finite element method is conceptually a physical rather than a mathematical approximation. The flow region is subdivided into a number of elements and permeabilities are specified for each element. Boundary conditions are specified in terms of heads and flow rates and a system of equations is solved to compute gradients and velocities in each element (Desai and Abel 1972 and Desai 1977). Two- and three-dimensional finite element seepage computer programs for both confined and unconfined flow problems have been developed at WES. Steady-state and transient problems (that can be treated as a series of steady-state problems) can be solved (Tracy 1973a; Tracy 1973b; and Hall, Tracy, and Radhakrishnan 1975). An interactive graphics preprocessor is available to generate the finite element grid (Tracy 1977a). It is possible to compute the stream function and potential and plot contours of these values to obtain the flow net (Christian 1980 and Christian 1980). Details concerning the selection of spatial and time meshes, computer time required, convergence, and stability are available (Desai 1977). Also, an interactive graphics postprocessor is available to assist in the analyses of the finite element results (Tracy...
A listing of finite element seepage computer programs used within the Corps is available (Edris and Vanadit-Ellis 1982).

b. Advantages. The finite element method is well suited to complex geometry, including sloping layers and pockets of materials of varying permeability. By varying the size of the elements, zones where seepage gradient or velocity is high can be accurately modeled.

c. Disadvantages. The finite element method is usually more costly than the finite difference method for simple problems (Rushton and Redshaw 1979).

d. Applications. The finite element method has been used in several cases to provide solutions to seepage problems.

(1) WES studies. As discussed previously, the finite element method was used at WES to simulate seepage conditions in streambanks induced by sudden drawdown of the river level. This study included a viscous flow model, field observations, and application of the finite difference and finite element methods. The results of the study indicated that the finite element method provided satisfactory solutions for transient unconfined fluid flow in porous, anisotropic, and nonhomogeneous media (Desai 1970 and Desai 1973).

(2) Location of phreatic surface. The finite element method has been used to determine the location of the phreatic surface in earth dams (Isaacs 1979; Isaacs 1980; Wei and Shieh 1979; and Desai and Kuppusamy 1980). The finite element method was used to locate the phreatic surface within tailings pond embankments and to define the subsurface flow of water from the pond. Results predicted using the finite element model were confirmed with measurements in a laboratory model and in the field (Kealy and Busch 1971).

(3) W.A.C. Bennett Dam. The finite element method was used to assess the potential seepage flows and uplift pressures in the foundation rock for W.A.C. Bennett Dam in British Columbia, Canada (see figure B-21). The finite element analysis (see figure B-22) was carried out assuming the following conditions:

(a) With an effective grout curtain.

(b) Without an effective grout curtain.

(c) With a drainage system.

(d) Without a drainage system.

(e) With various rock permeabilities.

The results of the finite element analysis, shown in figure B-23, indicate the greatest reduction in seepage flow and hydrostatic pressure could be accomplished by an effective grout curtain and downstream-drainage system (Taylor and Chow 1976).
(4) Corps of Engineers levees. The finite element method was used by the U. S. Army Engineer District, Rock Island, to study hydraulic sand fill levees along the Mississippi River (Schwartz 1976). Finite element and gradient plane\(^{(1)}\) analyses were used in conjunction with data from a full scale test levee to establish the material properties of the sand levees and to determine the exit point of the free seepage surface, the quantity of through seepage, and the exit gradients along the free discharge face. A parameter study was performed and dimensionless design charts were developed.

(5) Reservoir loading conditions on zoned embankments. The use of the finite element method to study the effect of initial filling of the reservoir, steady seepage, and rapid drawdown of the reservoir on zoned embankments has been given (Eisenstein 1979).

(6) Bureau of Reclamation dams. The Bureau of Reclamation has utilized two- and three-dimensional finite element methods, electrical analogy, and mathematical methods to analyze seepage flow through a dam embankment and

\[^{(1)}\] The gradient plane method is a graphical solution by means of the hodograph (see description by Casagrande 1937).
Figure B-22. Finite element study results (courtesy of International Commission on Large Dams\textsuperscript{269})

Figure B-23. Uplift pressure under various conditions (courtesy of International Commission on Large Dams\textsuperscript{269})
foundation (Mantei and Harris 1979). Narrows Dam, Colorado, on the South Platte River, was analyzed for seepage at the feasibility stage. Because of a pervious foundation, the planners called for a positive vertical cutoff by constructing a slurry wall down to the underlying shale. However, near the right abutment the shale drops away to depths too great for economical slurry wall construction. A three-dimensional finite element model (see figure B-24) was used to determine the vertical exit gradients at the downstream toe of the dam. The finite element model was used to study the effect of a toe drain, partial depth slurry trench, partially and fully penetrating relief wells (see figure B-25). Calamus Dam, Nebraska, on the Calamus River, was also analyzed by the Bureau of Reclamation for seepage at the feasibility stage (Mantei and Harris 1979; Mantei, Esmiol, and Cobb 1980; Mantei and Cobb 1981; and Cobb 1984). Calamus Dam has a setting very similar to Narrows Dam in the sense that it is an earth dam on a pervious foundation. However, the underlying shale at Calamus Dam is at such a great depth that it cannot be used as the base for a cutoff wall as it was for Narrows Dam. Early thinking on the project involved the use of a slurry trench cutoff down to a pervious sandstone formation. A three-dimensional finite element model (see figure B-26) was used to determine the effects of an embankment toe drain, slurry trench under upstream blanket, and/or relief wells at the downstream toe of the dam on the seepage rates and hydraulic gradients in the dam foundation. Time and expense in operating the large three-dimensional finite element models made it necessary that priority be given to studying the various design alternatives using the best estimate of permeability for each foundation material rather than conducting sensitivity studies to establish the effect of varying the permeability (see paragraph B-1). The three-dimensional finite element models were five elements deep, with the bottom layer of elements representing the sandstone, the next layer sand and gravel, recent alluvium, interbedded fine sand, and dune sand. A detailed three-dimensional finite element model was made for the outlet works area that defined more of the design details, such as the filter blanket under the stilling basin and channel and water table elevation controls, to study the effectiveness of relief wells around the stilling basin.

(7) Corps of Engineer dams. The finite element method was used by the U. S. Army Engineer District, Huntington, (1) in a reanalysis of the underseepage at Bolivar Dam, Ohio, completed in 1938 on Sandy Creek (U. S. Army Engineer District, Huntington 1977a). A two-dimensional finite element model (see figure B-27) was used to determine the effects of an embankment toe drain, upstream impervious blanket, and proposed relief wells on seepage quantities, exit gradients, and uplift pressures. A sensitivity study was conducted using the two-dimensional finite element model to determine the influence of various pool and rock surface levels, the permeability ratio of foundation soils, the existence of a downstream gravel layer, and the effective source of seepage entry upon underseepage. Typical test results for one set of boundary conditions and permeability values are given in figure B-28. Additional applications of the two-dimensional finite element method to conduct sensitivity analysis to assess the effect of permeability anisotropy and various seepage control measures was given by Lefebvre and coworkers (Lefebvre, Part, and

(1) Work was performed by Soil Testing Services, Inc., Northbrook, Illinois.
Figure B-24. Contours of exit gradient from three-dimensional finite element model study of Narrows Dam, Colorado (courtesy of American Society of Civil Engineers).
Figure B-25. Vertical exit gradients from three-dimensional finite element model study of Narrows Dam, Colorado (courtesy of American Society of Civil Engineers)
Figure B-26. Grid for three-dimensional finite element model study of Calamus Dam, Nebraska (courtesy of American Society of Civil Engineers)
Figure B-27. Grid for two-dimensional finite element model study of Bolivar Dam, Ohio (from U. S. Army Engineer District, Huntington)
Figure B-28. Results from two-dimensional finite element model study of Bolivar Dam, Ohio (from U. S. Army Engineer District, Huntington).
The finite element method was used by the U. S. Army Engineer District, Huntington, (1) in a reanalysis of the underseepage at Mohawk Dam, Ohio, completed in 1937 on the Walhonding River (U. S. Army Engineer District, Huntington 1979b). A three-dimensional finite element model (see figure B-29) was used to study the cause of unusually high relief well flows. Typical test results for one set of boundary conditions are given in figure B-30.

(1) Work was performed by Soil Testing Services, Inc., Northbrook, Illinois.
Figure B-29. Grid for three-dimensional finite element model study of Mohawk Dam, Ohio (from U. S. Army Engineer District, Huntington 89)
Figure B-30. Results from three-dimensional finite element model study of Mohawk Dam, Ohio (from U. S. Army Engineer District, Huntington).
APPENDIX C

ANALYSIS OF PRESSURE INJECTION TESTS (Ziegler 1976)

C-1. **Water Pressure Tests**

a. Water pressure tests are conducted by pumping water into a borehole at a constant pressure and measuring the flow rate. Water enters the rock mass along the entire length of borehole or along a test section (i.e., borehole interval) sealed off by one or more packers as shown in figure C-1. The test is rapid and simple to conduct and by conducting tests within intervals along the entire length of borehole, a permeability profile can be obtained.

b. In most pressure tests the water injection pressure is limited to a value which is not expected to produce an increase in the fracture width. An increase in the fracture width will cause erroneously high flow rates resulting in higher permeabilities than actually exist. A common criterion is to limit the water injection pressure to 1 psi/ft of borehole depth above the water table and 0.57 psi/ft of borehole depth below the water table. This criterion results in a maximum injection pressure less than the effective overburden pressure if the overburden has a unit weight greater than 144 lb/ft$^3$.

c. The coefficient of permeability, based upon laminar flow, is computed for a vertical test section with the following assumptions:

1. Medium is homogeneous and isotropic.
2. Laminar flow governed by Darcy's law.
3. Radial flow from a cylindrical and vertical borehole test section length, $\ell$. Radial flow implies that the equipotential surfaces form cylinders symmetrical about the axis of the borehole test section.
4. No inertia effects.
5. Boundary conditions (fig. C-2).
   a. At $r = r_o$, $H = H_o$.
   b. At $r = R$, $H = 0$.

Where

$r = \text{radial distance from the test section (L)}$

$r_o = \text{radius of borehole (L)}$

$H = \text{excess pressure head (L)}$

$H_o = \text{excess pressure head at the center of the test section (L) (either measured within the test section, or computed allowing for frictional head losses within the drill pipe)}$
R = radius of influence: radial distance from the test section corresponding to a 100 percent loss in excess head, $H_0 (L)$

Figure C-1. Typical water pressure test setups (prepared by WES)
Figure C-2. Homogeneous isotropic material--radial flow (prepared by WES)

d. Darcy's equation may be written

\[ v = k_e \frac{dh}{dr} \]

where

- \( v \) = flow velocity (L/T)
- \( k_e \) = laminar equivalent permeability (L/T)
- \( \frac{dh}{dr} \) = hydraulic gradient in the radial flow system (L/L)

e. The volume flow rate from the borehole cavity is

\[ Q = k_e \frac{dh}{dr} A \]

where

- \( Q \) = volume flow rate (L³/T)
$A = \text{area of an equipotential surface (} L^2\text{)}$

The area $A$ in the radial system is

$$A = 2\pi r l$$

Thus

$$Q = k_e 2\pi r l \frac{dh}{dr}$$

which can be written

$$\frac{dr}{r} = \frac{2\pi l}{Q} k_e dh$$

and integrated

$$\int_{r_o}^{R} \frac{dr}{r} = \frac{2\pi l k_e}{Q} \int_{H_0}^{Q} dh \quad \ldots (C-1)$$

$$\ln \frac{R}{r_o} = \frac{-2\pi l k_e}{Q} H_0$$

$$-Q = \frac{2\pi l k_e H_0}{\ln \left(\frac{R}{r_o}\right)}$$

The negative sign can be dropped by choosing flow away from the borehole as positive, thus

$$Q = \frac{2\pi l k_e H_0}{\ln \left(\frac{R}{r_o}\right)}$$

and the permeability is

C-4
f. Before equation C-2 can be applied, however, the radius of influence, $R$, must be determined or estimated. A rough estimate of $R$ is normally adequate since large variations in $R$ generally produce only small variations in the computed permeability. In equation C-2 the permeability is directly related to the term $\ln \left( \frac{R}{r_0} \right)$. The rate of change of $\ln \left( \frac{R}{r_0} \right)$, and thus permeability, decreases rapidly as $R$ increases. This is illustrated by considering the analysis of a pressure test conducted in an NX-size borehole. The increase in permeability, $k_e$ (based on equation C-2), as $R$ is increased from 1 ft to $10^6$ ft is shown below:

<table>
<thead>
<tr>
<th>Radius of Influence, $R$, ft</th>
<th>Equivalent Permeability, $k_e$, ft/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.1 $k_e$</td>
</tr>
<tr>
<td>10</td>
<td>3.2 $k_e$</td>
</tr>
<tr>
<td>100</td>
<td>4.3 $k_e$</td>
</tr>
<tr>
<td>1,000</td>
<td>5.4 $k_e$</td>
</tr>
<tr>
<td>10,000</td>
<td>6.5 $k_e$</td>
</tr>
<tr>
<td>100,000</td>
<td>7.5 $k_e$</td>
</tr>
<tr>
<td>1,000,000</td>
<td></td>
</tr>
</tbody>
</table>

Thus as $R$ is increased from 1 ft to $10^6$ ft, the computed permeability increases by less than one order of magnitude. To eliminate the arbitrary choice of $R$ as a source of error in the permeability calculation, it has been suggested that during a pressure test, pressure in the surrounding mass be observed in a nearby borehole. By changing the upper limits of integration in equation C-1 to correspond to an arbitrary distance $r$, within the boundaries $r_o$ to $R$ (fig. C-2) the following equations are developed:

\[
\int_{r_o}^{r} \frac{dr}{r} = \frac{2\pi k_e}{Q} \int_{h_o}^{H} dh
\]

Integration yields
where $H_1$ = excess pressure head at an arbitrary distance, $r_1$, from the test section (L) which can be written as

$$H_1 = H_0 - \frac{Q}{2\pi L k_e} \ln \left(\frac{r_1}{r_o}\right)$$

The coefficient of permeability, based upon nonlinear or turbulent flow, is computed for a vertical test section in a homogeneous, isotropic medium with the following assumptions:

1. Medium is homogeneous and isotropic.
2. Nonlinear or turbulent flow governed by the Missbach law
3. Radial flow from cylindrical and vertical borehole test section of length, $L$.
4. No inertia effects.
5. Boundary conditions (fig. C-2).
   a. At $r = r_o$, $H = H_o$.
   b. At $r = R$, $H = 0$.

h. The Missbach law may be written

$$v^m = k'_e \frac{dh}{dr}$$

where

- $m$ = degree of nonlinearity (generally between 1 and 2)
- $k'_e$ = turbulent equivalent permeability $(L/T)^m$

Volume flow rate for radial flow from the cavity is

$$Q = 2\pi r L v$$

C-6
and it follows that:

\[ Q^m = (2\pi \ell)^m v^m \]

\[ Q^m = (2\pi \ell)^m k_e^m \frac{dh}{dr} \]

\[ \frac{Q^m}{(2\pi \ell)^m} = r^m k_e^m \frac{dh}{dr} \]

\[ \frac{dr}{r^m} = k_e^m \left( \frac{2\pi \ell}{Q} \right)^m dh \]

\[ \int_{r_o}^{R} r^{-m} dr = k_e^m \left( \frac{2\pi \ell}{Q} \right)^m Q \int_{H_0}^{Q} dh \]

\[ \left( \frac{1}{1 - m} \right) \left( R^{1-m} - r_o^{1-m} \right) = -k_e^m \frac{(2\pi \ell)^m}{Q^m} H_0 \]

\[ -Q^m = k_e^m (2\pi \ell)^m H_0 (1 - m) \left( \frac{1}{R^{1-m} - r_o^{1-m}} \right) \]

Drop the negative sign by choosing flow away from the borehole as positive, yielding

\[ k_e^m = \frac{Q^m \left( R^{1-m} - r_o^{1-m} \right)}{(2\pi \ell)^m (1 - m) H_0} \]  \hspace{1cm} (C-4)

Equation C-4 can be rewritten as

\[ H_0 = E_1 Q^m \]  \hspace{1cm} (C-5)
where

\[ E_1 = \left( \frac{1}{k_e} \right) \left( \frac{1}{2\pi k} \right)^m \left( \frac{l}{l - m} \right) \left( R^{1-m} - r_o^{1-m} \right) \]

The logarithms of terms in equation C-5 yield the equation of a straight line:

\[ \log H_o = \log E_1 + m \log Q \]

A plot of \( \log H_o \) versus \( \log Q \) will be a straight line with an arithmetic slope equal to the degree of nonlinearity \( m \).

i. The coefficient of permeability for laminar flow in fissures is computed for a vertical test section with the following assumptions:

(1) Vertical borehole test section of length \( l \) is intersected by an arbitrary number of horizontal fissures.

(2) Fissures are constant aperture openings between smooth parallel plates.

(3) Radial flow occurs within each fissure and is governed by Darcy's law. No flow occurs in material between fissures.

(4) Each fissure has the same equivalent parallel plate aperture, \( e \).

(5) No inertia effects.

(6) Boundary conditions (fig. C-3).

(a) At \( r = r_o \), \( H = H_o \).

(b) At \( r = R \), \( H = 0 \).

j. The derivation of equations C-6 and C-7 proceeds in the same fashion as that given above for laminar flow through a homogeneous isotropic medium (equations C-2 and C-3). For the fissured medium the test section length, \( l \), is replaced by the quantity \( (ne) \) where \( n \) is the number of fissures intersecting the test section and \( e \) is the equivalent parallel plate aperture of each fissure. The resulting expression for the flow rate \( Q \) is

\[ Q = \frac{2\pi n e k_H H_o}{\ln (R/r_o)} \]
and the permeability of each fissure is

\[
k_j = \frac{Q \ln \left(\frac{R}{r_0}\right)}{2\pi ne H_o}
\]  

where \( k_j \) = laminar fissure permeability (L/T). The general expression for excess pressure head \( H_1 \) at an arbitrary distance \( r_1 \) between the boundaries \( r_0 \) and \( R \) is given by

\[
H_1 = H_o - \frac{Q}{2\pi ne k_j} \ln \frac{r_1}{r_0}
\]

which can be written as

\[
k_j = \frac{Q \ln \left(\frac{r_1}{r_0}\right)}{2\pi ne \left(H_o - H_1\right)}
\]
From the theory of viscous flow between smooth parallel plates

\[ k_j = \frac{e^2 \gamma_w}{12 \mu_w} \]  \hspace{1cm} (C-8)

where

\[ \gamma_w = \text{unit weight of water (F/L}^3) \]
\[ \mu_w = \text{dynamic viscosity of water (FT/L}^2) \]

Equation C-8 is substituted into equation C-6 to solve for the equivalent parallel plate aperture, \( e \):

\[ e = \left[ \frac{Q \ln \left( \frac{R}{r_o} \right)}{2 \pi n H_o} \left( \frac{12 \mu_w}{\gamma_w} \right) \right] \]

\( k \). The coefficient of permeability for nonlinear or turbulent flow in fissures is computed for a vertical test section with the following assumptions:

1. Vertical borehole test section of length \( \ell \) intersected by an arbitrary number of horizontal fissures.
2. Fissures are constant aperture openings between smooth parallel plates.
3. Radial flow occurs within each fissure and is governed by the Missbach law \( (\nu^n = k'i) \). No flows occur in material between fissures.
4. Each fissure has the same equivalent parallel plate aperture, \( e \).
5. No inertial effects.
6. Boundary conditions (fig. C-3).
   a. At \( r = r_o, H = H_o \).
   b. At \( r = R, H = 0 \).

1. The derivation of equation C-9 proceeds in the same fashion as that given above for nonlinear flow through a homogeneous isotropic medium (equation C-4). For the fissured medium, the test section length, \( \ell \), is replaced
by the quantity ne where n is the number of fissures intersecting the test section and e is the equivalent parallel plate aperture of each fissure. The resulting equation for the turbulent fissure permeability is

\[ k'_j = \frac{Q^m (R^{1-m} - r_o^{1-m})}{(2\pi ne)^m H_0 (1 - m)} \]  

(C-9)

where \( k'_j \) = turbulent fissure permeability \((L/T)^m\). Equation C-9 can be rewritten as

\[ H_0 = E_2 Q^m \] 

(C-10)

where

\[ E_2 = \left( \frac{1}{k'_j} \right) \left( \frac{1}{2\pi ne} \right)^m \left( \frac{1}{1 - m} \right) (R^{1-m} - r_o^{1-m}) \]

The logarithms of terms in equation C-10 yielded the equation of a straight line:

\[ \log H_0 = \log E_2 + m \log Q \]

A plot of \( \log H_0 \) versus \( \log Q \) will be a straight line with an arithmetic slope equal to the degree of nonlinearity, \( m \).

C-2. Air Pressure Tests.

a. Air pressure tests are conducted similarly to water pressure tests, with the essential difference being the replacement of water with air. The use of air, however, requires that permeability equations be modified for application to a compressible fluid and a conversion must be made from air to water permeability. To compute the permeability, flow is assumed to be laminar and governed by Darcy's law. The material tested is assumed to be a homogeneous isotropic porous medium. Darcy's law can be written

\[ v = k_i \]
where

\[ v = \text{flow velocity (L/T)} \]

\[ k = \text{coefficient of permeability (L/T)} \]

\[ i = \text{hydraulic gradient (L/L)} \]

b. The coefficient of permeability, \( k \), is dependent on properties of the medium and fluid or gas. Since the test involves the flow of air and permeability relating to the flow of water is desired, it is convenient to solve for the intrinsic permeability, \( k \), which is characteristic of the medium alone. The coefficient of permeability and intrinsic permeability are interrelated (Davis and Dewiest 1966, and Muskat 1946). For the flow of water

\[
k_e = \frac{k \gamma_w}{\mu_w} \tag{C-11}
\]

where

\[ k_e = \text{laminar equivalent permeability (L/T)} \]

\[ \gamma_w = \text{unit weight of water (F/L}^3) \]

\[ \mu_w = \text{dynamic viscosity of water (F/}L^2) \]

c. The conversion of intrinsic permeability measured by an air test to the water permeability can be in error due to differences in gas and fluid flow phenomena i.e., the Klinkenberg Effect (Weeks 1978), or alteration of the material's physical properties caused by a chemical reaction with the fluid or gas (Davis and Dewiest 1966, and Lynch 1962). In sediments rich in clay minerals, water permeability calculated from air measurements may be overestimated by a factor of 100 (Davis and Dewiest 1966). This overestimation is caused by a hydration of clays during waterflow which does not occur during airflow. Test results should be applied with caution as more experience and studies are needed to determine the limitations of equation C-11.

d. Muskat (1946, p. 679) observed that the flow of incompressible liquids is easily modified for application to the flow of a compressible gas. Steady-state solutions for analyzing constant water pressure test results are generally written in the form

\[
k_e = \frac{Q}{\Delta H_o} [C] \tag{C-12}
\]
where

\[ Q = \text{volume flow rate (L}^3/\text{T}) \]

\[ L = \text{length of test section (L)} \]

\[ H_0 = \text{excess pressure head at the center of the test section (L)} \]

The term \([C]\) is dependent on assumed flow and boundary conditions. The following equation is commonly used in analyzing water pressure test results.

\[
k_e = \frac{Q}{2H_o} \frac{\ln \left( \frac{R}{r_o} \right)}{2\pi}
\]

\[ (C-12)\]

where

\[ R = \text{radius of influence (L)} \]

\[ r_o = \text{radius of borehole (L)} \]

After solving for \(Q\) and converting to intrinsic permeability, \(k\), equation \(C-12\) becomes

\[
Q = \left( \frac{2\pi \mu K}{\mu_w} \right) \left[ \frac{P_o}{\ln \left( \frac{R}{r_o} \right)} \right]
\]

\[ (C-13)\]

where \(P_o = \gamma_w H_o\) = excess pressure at the center of test section (F/L²) the equation \((C-13)\) is based-on the assumption that:

(1) The medium is a homogeneous and isotropic porous continuum.

(2) The flow emitting from a cylindrical section of borehole is laminar, and governed by Darcy's equation.

(3) The flow pattern is ellipsoidal and symmetrical about the axis of the borehole test section.

e. Equation \(C-13\) can be modified for analyzing air pressure test results by making similar assumptions and considering the fluid to be compressible. Before modification, equation \(C-13\) must be written in terms of absolute pressures. The pressure, \(P_o\), equals the absolute pressure in the test section, \(P_b\), minus atmospheric pressure, \(P_a\). Equation \(C-13\) becomes
f. Equation C-14 is modified to apply to the flow of gas by replacing the pressure term \((\bar{P}_b - \bar{P}_a)\) by the expression

\[
Q = \left(\frac{2\pi K}{\mu_w}\right) \left[\frac{\bar{P}_b - \bar{P}_a}{\ln (R/r_o)}\right]
\]  

(C-14)

where \(\gamma_o\) is a constant at any point in the flow and defined as

\[
\gamma_o = \frac{\rho g}{\bar{P}^M} = \frac{\gamma}{\bar{P}^M} = \text{a constant}
\]  

(C-16)

Substitution of expression C-15 into equation C-14 (replace \(\mu_w\) with \(\mu_a\)) yields the following expression for the weight flow rate of air:
where

\[ Q_{WF} = \frac{2\pi K Y_o (\bar{P}_b^{1+M} - \bar{P}_a^{1+M})}{\mu_a (1 + M) \ln (R/r_o)} \]  

**C-18**

\[ k = \left( \frac{Q_{WF}}{Y_b} \right) \frac{\mu_a \ln (R/r_o)}{\pi \ell} \left( \frac{\bar{P}_b}{\bar{P}_o^2 - \bar{P}_a^2} \right) \]  

**C-19**

---

**h.** For convenience, the value of M is assumed equal to 1 which corresponds to isothermal expansion of an ideal gas as it moves from the borehole through the medium (for adiabatic expansion, \( M < 1 \)). Since equation C-16 is valid at any point in the flow and \( M + 1 \), \( Y_o \) can be replaced by \( Y_b / \bar{P}_b \) where \( Y_b \) and \( \bar{P}_b \) are the unit weight of air and absolute pressure in the test section, respectively. By substituting \( M = 1 \) and \( Y_b / \bar{P}_b \) in equation C-18, the intrinsic permeability, \( k \), can be expressed as

**i.** The weight flow rate from the test section, \( Q_{WF} \), is constant along the flow path and can be determined at the manifold by applying equation C-17:
where

\[ Q_{WF} = Q_m \gamma_m \]

\[ Q_m = \text{volume flow rate at the manifold (L}^3/\text{T)} \]

\[ \gamma_m = \text{unit weight of air at the manifold (F/L}^3) \]

The value of \( \gamma_m \) is related to the pressure and temperature by the equation of state (Vennard 1965):

\[ \gamma_m = \frac{\overline{p}_m}{R_g T_m} \]

where

\[ \gamma_m = \text{unit weight of air at the manifold (lb}_r/\text{ft}^3) \]

\[ \overline{p}_m = \text{absolute pressure at the manifold (lb}_r/\text{ft}^2) \]

\[ R_g = \text{engineering gas constant for air (53.3 ft-lb}_r/\text{lb}_r\text{-degrees Rankine)} \]

\[ T_m = \text{absolute temperature at the manifold (degrees Rankine)} \]

j. The unit weight of air in the test section, \( \gamma_b \), is also computed from the equation of state:

\[ \gamma_b = \frac{\overline{p}_m}{R_g T_b} \]

where \( T_b = \text{absolute temperature in test section (degrees Rankine)} \). All the parameters needed to compute the intrinsic permeability, \( k \), by equation C-19 have now been determined. The equivalent permeability applicable to the flow of water, \( k_e \), is computed by substitution of \( k \) into equation C-11 yielding the following general equation:

C-16
where

\[ k_e = \left( \frac{Q_m}{\pi \xi} \right) \left( \frac{\mu_a}{\mu_w} \right) \left( \frac{\bar{T}_b}{\bar{T}_m} \frac{\bar{P}_m}{\bar{P}_b} \right) \ln \left( \frac{R}{r_o} \right) \]

\[ k_e = \text{laminar equivalent permeability (L/T)} \]

\[ Q_m = \text{volume flow rate at the manifold (L}^3/\text{T)} \]

\[ \xi = \text{length of the test section (L)} \]

\[ \gamma_w = \text{unit weight of water (F/L}^3) \]

\[ \mu_a = \text{dynamic viscosity of air (FT}^2/\text{L}^2) \]

\[ \mu_w = \text{dynamic viscosity of water (FT}^2/\text{L}^2) \]

\[ \bar{T}_b = \text{absolute temperature in test section (degrees Rankins)} \]

\[ \bar{P}_m = \text{absolute pressure at the manifold (F/L}^2) \]

\[ \bar{T}_m = \text{absolute temperature at the manifold (degrees Rankins)} \]

\[ \bar{P}_b = \text{absolute pressure in test section (F/L}^2) \]

\[ \bar{P}_a = \text{atmospheric pressure (F/L}^2) \]

\[ R = \text{radius of influence (L)} \]

\[ r_o = \text{radius of the borehole (L)} \]

C-3. **Pressure Holding Tests.**

a. The pressure holding test, sometimes called a pressure duration or pressure drop test, is conducted by pressurizing the test section to a known value, then stopping the water supply and observing the rate of pressure decay. A pressure holding test is usually conducted in conjunction with a water pressure test. For the conduct of the test, a pressure transducer mounted in the test section is used to provide a continuous record of pressure versus time.

b. Tests in fissured rock can be analyzed through application of the parallel plate analogy. The laminar fissure permeability, \( k_j \), is related to the pressure drop data by the expression
where

\[ r_o = \text{radius of borehole (L)} \]
\[ H_{o1} = \text{excess pressure head at the center of the test section at the initiation of a pressure drop test (L)} \]
\[ H_{o2} = \text{excess pressure head at the center of the test section at the completion of a pressure drop test (L)} \]
\[ R = \text{radius of influence (estimated) (L)} \]
\[ n = \text{number of fissures intersecting the test section} \]
\[ t_{\text{drop}} = \text{duration of the pressure drop test (T)} \]

Equation C-20 was derived by Maini (1971) based on the following assumptions.

1. Radial flow occurs from a vertical test section and is governed by Darcy's law.
2. Fissure system intersecting the test section is represented by horizontal parallel plates of equal aperture and spacing.
3. Test zone is saturated. Maini (1971) suggests that before conducting tests in zones above the ground-water table, water be pumped into the borehole test section to saturate the fissure system in the immediate vicinity of the borehole.

d. The equivalent parallel aperture, \( e \), in equation C-20 is unknown; however, by parallel plate analogy

\[ k_j = \frac{e^2 \gamma_w}{12 \mu_w} \]  
\[ \text{(C-21)} \]

which when substituted into equation C-20 yields

\[ e^3 = \frac{r_o^2 \ln \left( \frac{H_{o1}}{H_{o2}} \right) \ln \left( \frac{R}{r_o} \right)}{2n \gamma_w t_{\text{drop}}} 12 \mu_w \]  
\[ \text{(C-22)} \]
The aperture, \( e \), is computed by equation C-22 and substituted into equation C-21 to obtain the permeability \( k_j \). If the test zone is modeled as a homogeneous isotropic porous medium and assumptions (1) and (3) above are valid, an expression for the laminar equivalent permeability, \( k_e \), is obtained by replacing the quantity \( n e \) in equation C-20 with the length of the test section, \( l \), to yield

\[
 k_e = \frac{r_o^2 \ln\left(\frac{H_1}{H_o}\right) - \ln(R/r_o)}{2l(t_{\text{drop}})}
\]

The pressure drop test is a suitable supplement to the water pressure test and possesses certain advantages. The pressure drop test is likely to require a significantly smaller volume of water than that needed in a constant pressure test. The savings in water can be important when conducting tests in regions with a limited water supply. The measurement of pressure in the test section by an electric transducer allows the pressure drop test to be conducted with low initial pressures regardless of the depth of the test section, since the water level in the flow pipe only needs to be above the top of the test section to initiate a test. The use of low initial pressures has the additional advantage of reducing friction pressure losses during the conduct of a test.
APPENDIX D
FILTER DESIGN

D-1. General. The objective of filters and drains used as seepage control measures for embankments is to efficiently control the movement of water within and about the embankment. In order to meet this objective, filters and drains must, for the project life and with minimum maintenance, retain the protected materials, allow relatively free movement of water, and have sufficient discharge capacity. For design, these three necessities are termed, respectively, piping or stability requirement, permeability requirement, and discharge capacity. This appendix will explain how these requirements are met for cohesionless and cohesive materials, and provide general construction guidance for installation of filters and drains. The terms filters and drains are sometimes used interchangeably. Some definitions classify filters and drains by function. In this case, filters must retain the protected soil and have a permeability greater than the protected soil but do not need to have a particular flow or drainage capacity since flow will be perpendicular to the interface between the protected soil and filter. Drains, however, while meeting the requirements of filters, must have an adequate discharge capacity since drains collect seepage and conduct it to a discharge point or area. In practice, the critical element is not definition, but recognition, by the designer, when a drain must collect and conduct water. In this case the drain must be properly designed for the expected flows. Where it is not possible to meet the criteria of this appendix, the design must be cautiously done and based on carefully controlled laboratory filter tests (Perry 1985).

D-2. Stability. Filters and drains(1) allow seepage to move out of a protected soil more quickly than the seepage moves within the protected soil. Thus, the filter material must be more open and have a larger grain size than the protected soil. Seepage from the finer soil to the filter can cause movement of the finer soil particles from the protected soil into and through the filter. This movement will endanger the embankment. Drains, however, while meeting the requirements of filters, must have an adequate discharge capacity since drains collect seepage and conduct it to a discharge point or area. In practice, the critical element is not definition, but recognition, by the designer, when a drain must collect and conduct water. In this case the drain must be properly designed for the expected flows. Where it is not possible to meet the criteria of this appendix, the design must be cautiously done and based on carefully controlled laboratory filter tests (Perry 1985).

---

(1) In paragraphs D-2 and D-3 the criteria apply to drains and filters; for brevity, only the word filter will be used.

(2) In practice, it is normal for a small amount of protected soil to move into the filter upon initiation of seepage. This action should quickly stop and may not be observed when seepage first occurs. This is one reason that initial operation of embankment seepage control measures should be closely observed by qualified personnel.
water from the protected soil. Criteria developed by many years of experience are used to design filters and drains which will prevent the movement of protected soil into the filter. This criterion, called piping or stability criterion, is based on the grain-size relationship between the protected soil and the filter. In the following, the small character "d" is used to represent the grain size for the protected (or base) material and the large character "D" the grain size for the filter material.

Determine filter gradation limits using the following steps:\(^{(3)}\)

1. Determine the gradation curve (grain-size distribution) of the base soil material. Use enough samples to define the range of grain sizes for the base soil or soils and design the filter gradation based on the base soil that requires the smallest $D_{15}$ size.

2. Proceed to step 4 if the base soil contains no gravel (material larger than No. 4 sieve).

3. Prepare adjusted gradation curves for base soils with particles larger than the No. 4 (4.75 mm) sieve.
   a. Obtain a correction factor by dividing 100 by the percent passing the No. 4 (4.75 mm) sieve.
   b. Multiply the percentage passing each sieve size of the base soil smaller than No. 4 (4.75 mm) by the correction factor from step 3a.
   c. Plot these adjusted percentages to obtain a new gradation curve.
   d. Use the adjusted curve to determine the percent passing the No. 200 (0.075 mm) sieve in step 4.

4. Place the base soil in a category based on the percent passing the No. 200 (0.075 mm) sieve in accordance with Table D-1.

5. Determine the maximum $D_{15}$ size for the filter in accordance with Table D-2. Note that the maximum $D_{15}$ is not required to be smaller than 0.20 mm.

6. To ensure sufficient permeability, set the minimum $D_{15}$ greater than or equal to 3 to 5 x maximum $d_{15}$ of the base soil before regrading but no less than 0.1 mm.

Table D-1. Categories of Base Soil Materials

<table>
<thead>
<tr>
<th>Category</th>
<th>Percent Finer Than the No. 200 (0.075 mm) Sieve</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>&gt;85</td>
</tr>
<tr>
<td>2</td>
<td>40-85</td>
</tr>
<tr>
<td>3</td>
<td>15-39</td>
</tr>
<tr>
<td>4</td>
<td>&lt;15</td>
</tr>
</tbody>
</table>

7. Set the maximum particle size at 3 in. (75 mm) and the maximum passing the No. 200 (0.075 mm) sieve at 5 percent. The portion of the filter material passing the No. 40 (0.425 mm) sieve must have a plasticity index (PI) of zero when tested in accordance with EM 1110-2-1906, "Laboratory Soils Testing."

8. Design the filter limits within the maximum and minimum values determined in steps 5, 6, and 7. Standard gradations may be used if desired. Plot the limit values, and connect all the minimum and maximum points with straight lines. To minimize segregation and related effects, filters should have relatively uniform grain-size distribution curves, without "gap grading"—sharp breaks in curvature indicating absence of certain particle sizes. This may require setting limits that reduce the broadness of filters within the maximum and minimum values determined. Sand filters with \( D_{90} \) less than about 20 mm generally do not need limitations on filter broadness to prevent segregation. For coarser filters and gravel zones that serve both as filters and drains, the ratio \( D_{90}/D_{10} \) should decrease rapidly with increasing \( D_{10} \) size. The limits in Table D-3 are suggested for preventing segregation during construction of these coarser filters.
Table D-2. Criteria for Filters

<table>
<thead>
<tr>
<th>Base Soil Category</th>
<th>Base Soil Description, and Percent Finer Than No. 200 (0.075 mm) Sieve (a)</th>
<th>Filter Criteria (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fine silts and clays; more than 85% finer</td>
<td>(c) $D_{15} \leq 9 \times d_{85}$</td>
</tr>
<tr>
<td>2</td>
<td>Sands, silts, clays, and silty and clayey sands; 40 to 85% finer</td>
<td>$D_{15} \leq 0.7 \text{ mm}$</td>
</tr>
</tbody>
</table>
| 3                  | Silty and clayey sands and gravels; 15 to 39% finer                     | (d), (e) $D_{15} \leq \left(\frac{40-A}{40-15}\right)$  
                             $[(4 \times d_{85}) - 0.7 \text{ mm}] + 0.7 \text{ mm}$ |
| 4                  | Sands and gravels; less than 15% finer                                  | (f) $D_{15} \leq 4 \text{ to } 5 \times d_{85}$ |

(a) Category designation for soil containing particles larger than 4.75 mm is determined from a gradation curve of the base soil which has been adjusted to 100% passing the No. 4 (4.75 mm) sieve.

(b) Filters are to have a maximum particle size of 3 in. (75 mm) and a maximum of 5% passing the No. 200 (0.075 mm) sieve with the plasticity index (PI) of the fines equal to zero. PI is determined on the material passing the No. 40 (0.425 mm) sieve in accordance with EM 1110-2-1906, "Laboratory Soils Testing." To ensure sufficient permeability, filters are to have a $D_{15}$ size equal to or greater than $4 \times d_{15}$ but no smaller than 0.1 mm.

(c) When $9 \times d_{85}$ is less than 0.2 mm, use 0.2 mm.

(d) $A$ = percent passing the No. 200 (0.075 mm) sieve after any regrading.

(e) When $4 \times d_{85}$ is less than 0.7 mm, use 0.7 mm.

(f) In category 4, the $D_{15} \leq 4 \times d_{85}$ criterion should be used in the case of filters beneath riprap subject to wave action and drains which may be subject to violent surging and/or vibration.
Table D-3. $D_{10}$ and $D_{90}$ Limits for Preventing Segregation

<table>
<thead>
<tr>
<th>Minimum $D_{10}$ (mm)</th>
<th>Maximum $D_{90}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.5</td>
<td>20</td>
</tr>
<tr>
<td>0.5 - 1.0</td>
<td>25</td>
</tr>
<tr>
<td>1.0 - 2.0</td>
<td>30</td>
</tr>
<tr>
<td>2.0 - 5.0</td>
<td>40</td>
</tr>
<tr>
<td>5.0 - 10</td>
<td>50</td>
</tr>
<tr>
<td>10 - 50</td>
<td>60</td>
</tr>
</tbody>
</table>

D-3. **Permeability.** The requirement that seepage move more quickly through the filter than through the protected soil (called the permeability criterion) is again met by a grain-size relationship criterion based on experience:

\[
\text{Permeability} = \frac{15 \text{ percent size of filter material}}{15 \text{ percent of the protected soil}} \geq 3 \text{ to } 5 \quad \text{(D-1)}
\]

Permeability of a granular soil is roughly proportional to the square of the 10- to 15-percent size material. Thus, the permeability criterion ensures that filter materials have approximately 9 to 25 or more times the permeability of the protected soil. Generally, a permeability ratio of at least 5 is preferred; however, in the case of a wide band of uniform base material gradations, a permeability ratio as low as 3 may be used with respect to the maximum 15-percent size of the base material. There may be situations, particularly where the filter is not part of a drain, where the permeability of the filter is not important. In those situations, this criterion may be ignored.

D-4. **Applicability.** The previously given filter criteria in Table D-2 and Equation D-1 are applicable for all soils (cohesionless or cohesive soils) including dispersive soils.(4) However, laboratory filter tests for dispersive soils, very fine silt, and very fine cohesive soils with high plastic limits are recommended.

---

D-5. **Perforated Pipe.** The following criteria are applicable for preventing infiltration of filter material into perforated pipe, screens, etc.:

\[
\text{Minimum 50 percent size of filter material} \geq 1.0
\]

In many instances a filter material meeting the criteria given by Table D-2 and Equation D-1 relative to the material being drained is too fine to meet the criteria given by Equation D-2. In these instances, multilayered or "graded" filters are required. In a graded filter each layer meets the requirements given by Table D-2 and Equation D-1 with respect to the pervious layer with the final layer in which a collector pipe is bedded also meeting the requirements given by Equation D-2. Graded filter systems may also be needed when transitioning from fine to coarse materials in a zoned embankment or where coarse material is required for improving the water-carrying capacity of the system.

D-6. **Gap-Graded Base.** The preceding criteria cannot, in most instances, be applied directly to protect severely gap- or skip-graded soils. In a gap-graded soil such as that shown in Figure D-1, the coarse material simply floats in the matrix of fines. Consequently, the scattered coarse particles will not deter the migration of fines as they do in a well-graded material. For such gap-graded soils, the filter should be designed to protect the fine matrix rather than the total range of particle sizes. This is illustrated in Figure D-1. The 85-percent size of the total sample is 5.2 mm. Considering only the matrix material, the 85-percent size would be 0.1 mm resulting in a much finer filter material being required. This procedure may also be followed in some instances where the material being drained has a very wide range of particle sizes (e.g., materials graded from coarse gravels to significant percentages of silt or clay). For major structures such a design should be checked with filter tests.

D-7. **Gap-Graded Filter.** A gap-graded filter material must never be specified or allowed since it will consist of either the coarse particles floating in the finer material or the fine material having no stability within the voids produced by the coarse material. In the former case the material may not be permeable enough to provide adequate drainage. The latter case is particularly dangerous since piping of the protected material can easily occur through the relatively large, loosely filled voids provided by the coarse material.

---

(5) EM 1110-2-2300 states, "Collector pipe should not be placed within the embankment, except at the downstream toe, because of the danger of either breakage or separation of joints, resulting from fill placement and compaction operations, or settlement, which might result in either clogging and/or piping."
Figure D-1. Analysis of gap-graded material (from EM 1110-2-1913)
D-8. **Broadly Graded Base.** One of the more common soils used for embankment dams is a broadly graded material with particle sizes ranging from clay sizes to coarse gravels and including all intermediate sizes. These soils may be of glacial, alluvial-colluvial, or weathered rock origin. As noted by Sherard, since the 85-percent size of the soil is commonly on the order of 20 to 30 mm, a direct application of the stability criteria $D_{15}/d_{85} \leq 4$ to 5 would allow very coarse uniform gravel without sand sizes as a downstream filter, which would not be satisfactory (Sherard 1979). The typical broadly graded soils fall in Soil Category 2 in Table D-2 and require a sand or gravelly filter with $D_{15} \leq 0.7$ mm.

D-9. **Example of Graded Filter Design for Drain.** Seldom, if ever, is a single gradation curve representative of a given material. A material is generally represented by a gradation band which encompasses all the individual gradation curves. Likewise, the required gradation for the filter material is also given as a band. The design of a graded filter which shows the application of the filter criteria where the gradations are represented by bands is illustrated in Figure D-2. A typical two-layer filter for protecting an impervious core of a dam is illustrated. The impervious core is a fat clay (CH) with a trace of sand which falls in Category 1 soil in Table D-2. The criterion $D_{15} \leq 9 \times d_{85}$ is applied and a "point a" is established in Figure D-2. Filter material graded within a band such as that shown for Filter A in Figure D-2 is acceptable based on the stability criteria. The fine limit of the band was arbitrarily drawn, and in this example, is intended to represent the gradation of a readily available material. A check is then made to ensure that the 15-percent size of the fine limit of the filter material band (point b) is equal to or greater than 3 to 5 times the 15-percent size of the coarse limit of the drained material band (point c). Filter A has a minimum $D_{10}$ size and a maximum $D_{90}$ size such that, based on Table D-3, segregation during placement can be prevented. Filter A meets both the stability and permeability requirements and is a suitable filter material for protecting the impervious core material. The second filter, Filter B, usually is needed to transition from a fine filter (Filter A) to coarse materials in a zoned embankment dam. Filter B must meet the criteria given by Table D-2 with respect to Filter A. For stability, the 15-percent size of the coarse limit of the gradation band for the second filter (point d) cannot be greater than 4 to 5 times the 85-percent size of the fine limit of the gradation band for Filter A (point e). For permeability, the 15-percent size of the fine limit (point f) must be at least 3 to 5 times greater than the 15-percent size of the coarse limit for Filter A (point a). With points d and f established, the fine and coarse limits for Filter B may be established by drawing curves through the points approximately parallel to the respective limits for Filter A. A check is then made to see that the ratio of maximum $D_{90}/D_{10}$ size of Filter B is approximately in the range as indicated in Table D-3. A well-graded filter which usually would not meet the requirements in Table D-3 may be used if segregation can be controlled during placement. Figure D-2 is intended to show only the principles of filter design. The design of thickness of a
## Figure D-2. Illustration of the design of a graded filter

<table>
<thead>
<tr>
<th>U.S. Standard Sieve Opening in Inches</th>
<th>U.S. Standard Sieve Numbers</th>
<th>Hydrometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 0</td>
<td>11 15 12 10 8 6 4 3 2 1 1/4 1 1/2 5/8 3/4</td>
<td>(d_{85}) min</td>
</tr>
<tr>
<td>10 7</td>
<td>4 3 2</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **Filter A**: \(a < 9 (d_{85})\) min
- **Filter B**: \(f > 3 \text{ to } 5a\)
- **Impervious Core**: \(b > 3 \text{ to } 5c\)
- **Filter C**: \(d \leq 4 \text{ to } 5e\)

**GRAIN SIZE IN MILLIMETERS**

**PERCENT FINEST BY WEIGHT**

**PERCENT COARSEST BY WEIGHT**
filter for sufficient seepage discharge capacity is done by applying Darcy's Law, \( Q = kia \), and one example is presented in Chapter 8 of the main text.

D-10. **Construction.** EM 1110-2-1911 and EM 1110-2-2300 provide guidance for construction. Major concerns during construction include:

a. Prevention of contamination of drains and filters by runoff containing sediment, dust, construction traffic, and mixing with nearby fine-grained materials during placement and compaction. Drain and filter material may be kept at an elevation higher than the surrounding fine-grained materials during construction to prevent contamination by sediment-carrying runoff.

b. Prevention of segregation, particularly well-graded filters, during handling and placement.

c. Proper in-place density is usually required to be an average of 85-percent relative density with no area less than 80-percent relative density. Granular materials containing little or no fines should be saturated during compaction to prevent "bulking" (low density) which can result in settlement when overburden materials are placed and the drain is subsequently saturated by seepage flows.

d. Gradation should be monitored closely so that designed filter criteria are met.

e. Thickness of layers should be monitored to ensure designed discharge capacity and continuity of the filter.

Thus, quality control/assurance is very important during filter construction because of the critical function of this relatively small part of the embankment.

D-11. **Monitoring.** Monitoring of seepage quantity and quality (see Chapter 13 for method of monitoring seepage) once the filter is functioning is very important to the safety of the embankment. An increase in seepage flow may be due to a higher reservoir level or may be caused by cracking or piping. The source of the additional seepage should be determined and action taken as required (Chapters 12, 13, and 14). Decreases in seepage flows may also signal dangers such as clogging of the drain(s) with piped material, iron oxide, calcareous material, and effects of remedial grouting. Again, the cause should be determined and appropriate remedial measures taken. Drain outlets should be kept free of sediment and vegetation. In cold climates, design or maintenance measures should be taken to prevent clogging of drain outlets by ice.

D-10